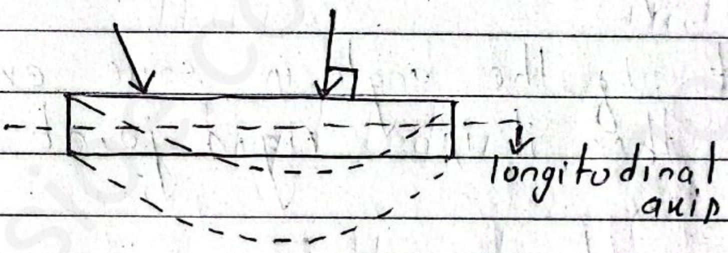
Chapter - 6 : Analysis of Beam & Frame :
(Analysis of Structure):

What is structure?

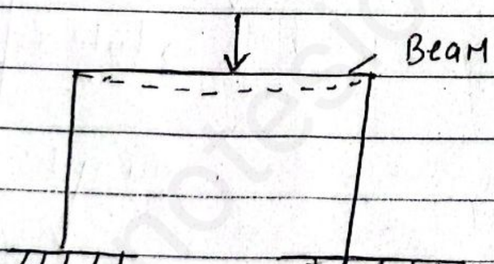
→ It is a combination of inter connected members designed to resist external forces such that it fulfills its intended purpose.

Beam :



→ One of the most common structural element.

→ It is the structural element that is subjected to bending by loads acting transverse to its longitudinal axis



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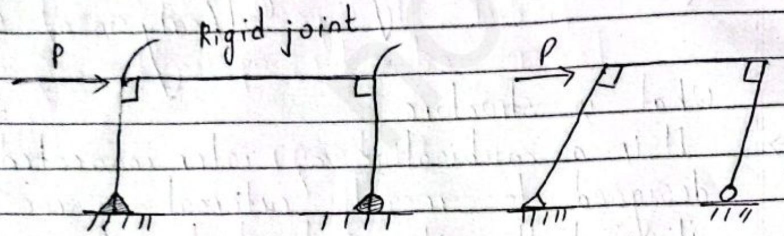
Frame:



A structure composed of members connected by rigid joints.

→ ~~bent~~ If the angle is same even after bent, then it is rigid joint

Rigid joint
The angle between the ends of various members at a joint remain unchanged as the frame deforms under load.

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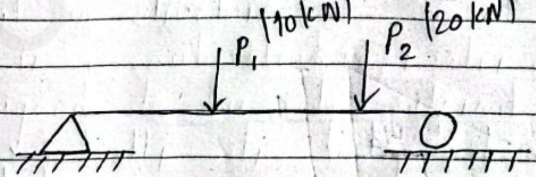
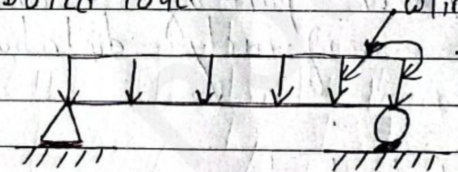
Various types of load on beam/frame:

① Concentrated load / Point load:



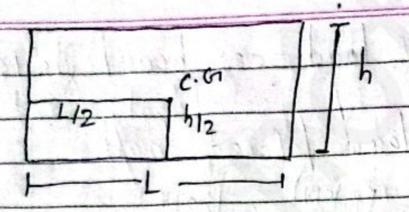
When a load is applied over a small area, it may be idealised as a concentrated load on point load.
It is expressed in units of force. Newton(N) kilonewton(kN)

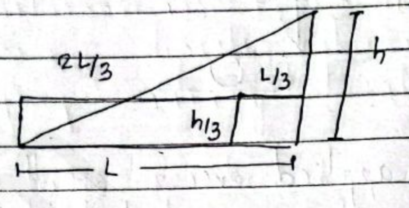
② Distributed load:



When a load is spread over the axis of beam, it is represented by distributed load. Distributed load is measured by its intensity.
Intensity is expressed in units of force per unit distance. (N/m) (kN/m).

Total load = Intensity × distance.
= 10 × 5 = 50N.

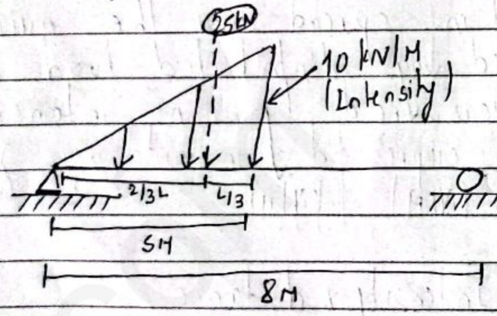




2 a) Uniformely diptributed load:

→ when the intensity 'w' per unit length has a constant value over part of beam, the load is said to be uniformely diptributed load

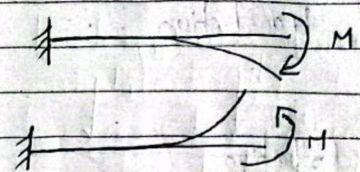
b) Varying diptributed load:



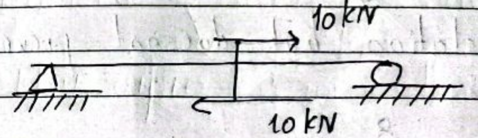
Total load = $\frac{1}{2} \times b \times h$

~~Linearly diptributed load.~~

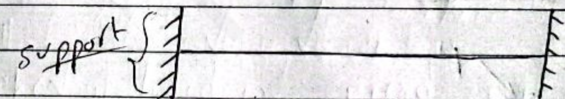
3) Moment:



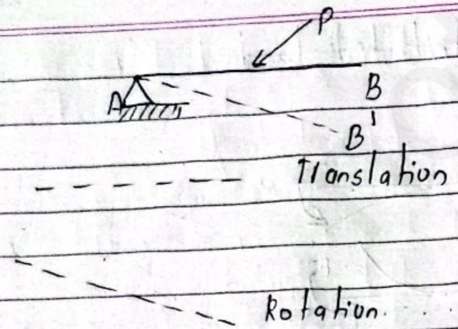
4) Couple:



Types of support & support idealization:



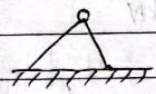
Structures are restrained so that they cannot move freely in space such restriction on the free motion of a body are called restraints and are provided by supports.

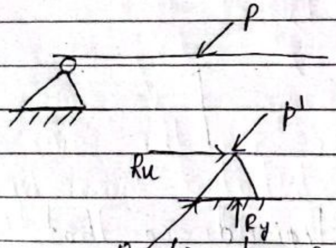


① **Pin support / Hinge support:**

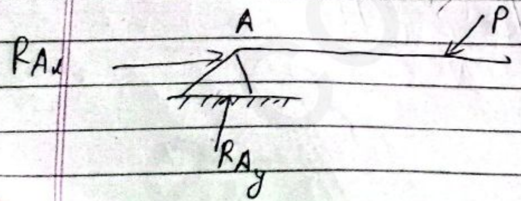
prevents translation but doesnot prevent rotation.

Symbol:

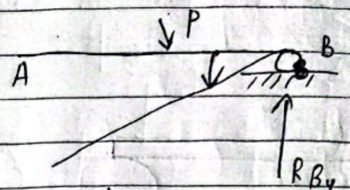




R (Reaction force exerted by support on beam)



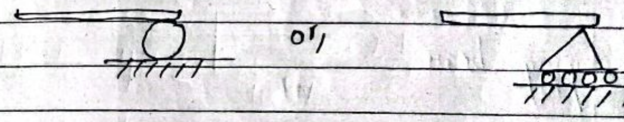
② **Roller support:**



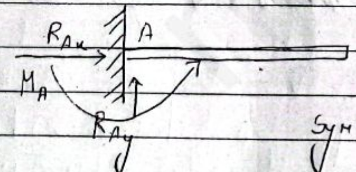
vertical direction resist

A roller support supplies resultant reaction force which acts normal to the surface on which rollers roll.

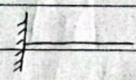
Symbol:



③ **Fixed support / clamp support:**



Symbol:



→ This kind of support prevents translation as well as rotation.

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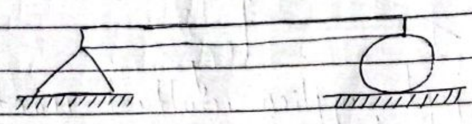
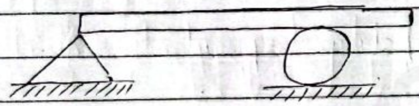
Types of beam on the basis of support and conditions:

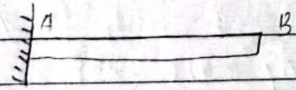
a) Simply supported beam:



b) Overhanging beam:



c) Cantilever beam:



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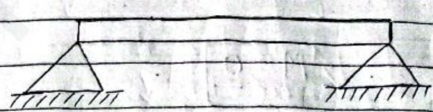
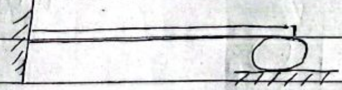
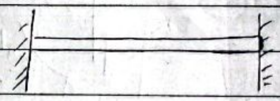
d) Hinged beam:



e) Fixed at one end and simply supported at other:



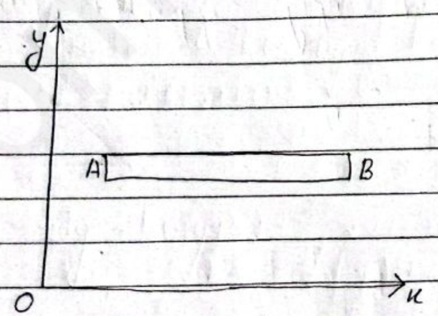
f) Fixed beam:



Equation of static equilibrium:

$$\Sigma F = 0 \qquad \Sigma M = 0$$

- Eq^n of static equilibrium for 2D structure (planar structure):



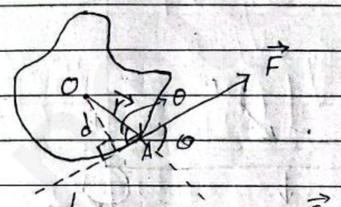
$\Sigma F_x = 0$ $\quad$ $\Sigma M_z = 0$

$\Sigma F_y = 0$

A body that is initially at rest and remains at rest when acted upon by a system of forces is said to be in state of static equilibrium.

For such a state to exist, it is necessary that the combined resultant effect of the system of forces shall be neither a force nor couple. Otherwise there will be a tendency for motion of body.

Moment of a force:



Let us consider a force $\vec{F}$ acting on a rigid body at point A. The position of A can be conveniently defined by the vector $\vec{r}$ which joins the fixed reference point O with A.

$\vec{r}$ is known as the position vector of A.

Moment of a force $\vec{F}$ about O is defined as the vector product of $\vec{r}$ and $\vec{F}$.

$$\vec{M}_O = \vec{r} \times \vec{F}$$
$$= rF\sin\theta$$

Statically indeterminate

→ Suppose we have 3 formulas & 4 problems.

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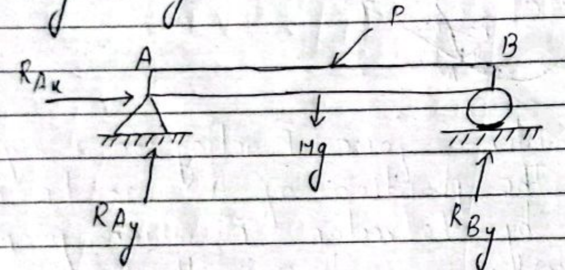
$M_o = Fd$ $(d = r\sin\theta)$

where, d is the perpendicular distance between O & line of action of force F.

The moment $\vec{M_o}$ is perpendicular to the plane containing $\vec{r}$ & $\vec{F}$.

The magnitude of $\vec{M_o}$ measures the tendency of force $\vec{F}$ to make the rigid body rotate about a fixed axis about O.

Free Body Diagram:



→ F·B·D are the basis of successful analysis of structure.

In solving a problem containing a rigid body in equilibrium, it is essential to consider all the forces acting on the body.

Statically determinate

→ Statics से problem determine गर्छ।
Suppose, we have 3 formulas & 3 problems.

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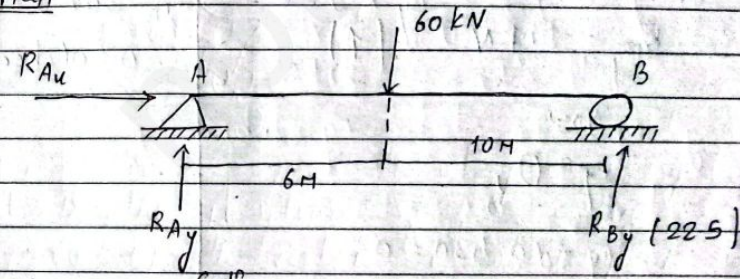
It is equally important to exclude any force that is not directly applied to the body.

Omitting a force or adding an extra one would destroy the conditions of equilibrium.

Computation of reactions:

Computation of reaction involves a straight forward application of equations of static equilibrium.

Numerical:



Sol^n

$\xrightarrow{+} \sum F_x = 0$

$R_{Ax} = 0$

$(\uparrow^+) \sum F_y = 0$

$R_{Ay} - 60 + R_{By} = 0$

Now,

$R_{Ay} - 60 + 22.5 = 0$

$\therefore R_{Ay} = 37.5\,kN$

$(\circlearrowleft^+) \sum M_A = 0$

$-60 \times 6 + R_{By} \times 16 = 0$

$\therefore R_{By} = 22.5\,kN$

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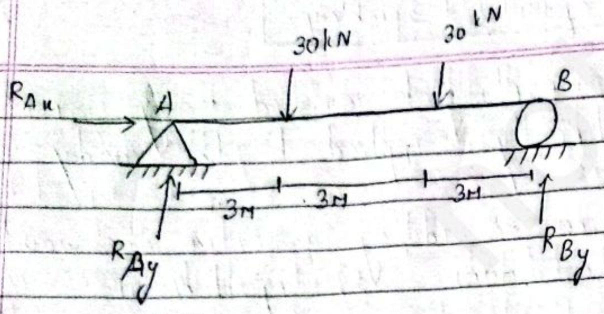

Sol^n

- $(\xrightarrow{+}) \sum F_x = 0$

$R_{Ax} = 0$

- $(\uparrow^+) \sum F_y = 0$

$R_{Ay} - 30 - 30 + R_{By} = 0$ — (1)

- $(\circlearrowleft^+) \sum M_A = 0$

or, $-30\times3 - 30\times6 + R_{By}\times9 = 0$

or, $-90 - 180 + R_{By}\times9 = 0$

$\therefore R_{By} = 30 kN\ (\uparrow)$

Now,

eq^n (1),

$R_{Ay} - 60 + 30 = 0$

$\therefore R_{Ay} = 30 kN\ (\uparrow)$

$(\circlearrowleft^+) \sum M_B = 0$

~~or, $-30\times3 - 30\times6 + R_{Ay}\times9 = 0$~~

or, $-30\times9 + 30\times6 + 30\times3 = 0$

$\therefore 0 = 0$ (check)

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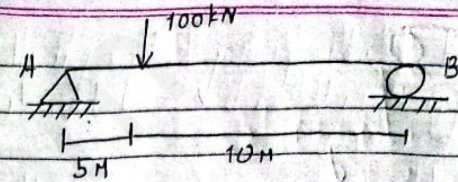
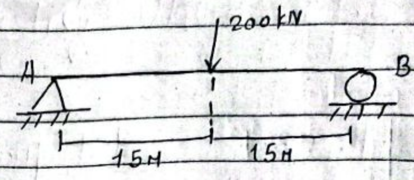
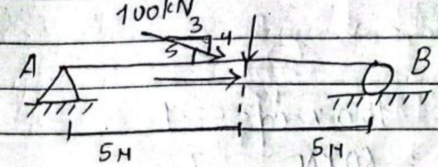
(I)



(II)



(III)



(I) Sol^n

- $(\xrightarrow{+}) \sum F_x = 0$

- $(\uparrow^+) \sum F_y = 0$

or, $R_{Ay} - 100 + R_{By} = 0$ — (1)

$(\circlearrowleft^+) \sum M_A = 0$

or, $-100\times5 + R_{By}\times15 = 0$

$\therefore R_{By} = 33.33 kN$

Now,

or, $R_{Ay} = 100 - 33.33$

$\therefore R_{Ay} = 66.67 kN$.

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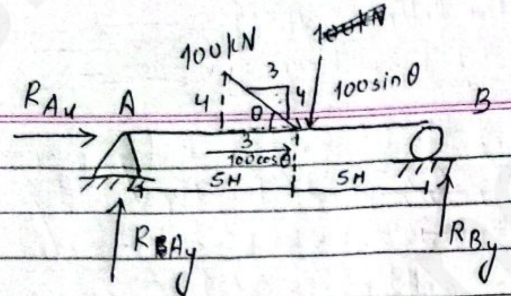
(iii)



Sol^n

$\tan\theta = \frac{P}{b}$

or, $\theta = \tan^{-1}\left|\frac{4}{3}\right| = 53.1°$

Now, we know,

for x-component,

$100\cos\theta = 100 \times \cos 53.1$
$= 60 kN$

For y-component,

$100\sin\theta = 100 \times \sin 53.1$
$= 80 kN$

Using eq^n of static equilibrium,

- $(\xrightarrow{+}) \Sigma F_x = 0$

$R_{Ax} + 60 = 0$

$\therefore R_{Ax} = -60 kN (\leftarrow)$

$(\circlearrowleft^+) \Sigma M_A = 0$

or, $-80 \times 5 + 10 \times R_{By} = 0$

$\therefore R_{By} = 40 kN$

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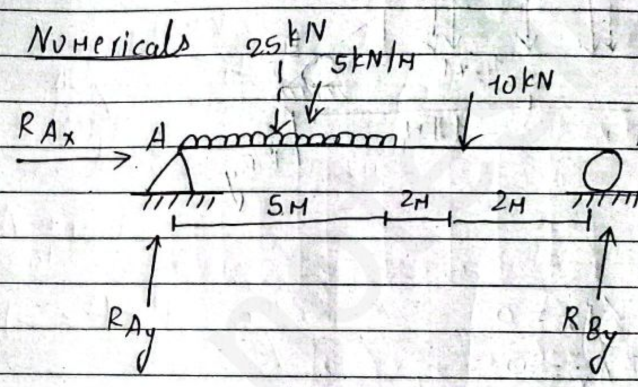
- $(\uparrow^+) \Sigma F_y = 0$

or, R_{Ay} ~~$- 60 +$~~ $- 80 + R_{By} = 0$

or, $R_{Ay} - 80 + 40 = 0$

$\therefore R_{Ay} = 40 kN$

- Numericals



Sol^n

- $(\xrightarrow{+}) \Sigma F_x = 0$

$R_{Ax} = 0$

$(\circlearrowleft^+) \Sigma M_A = 0$

or, $-25 \times 2.5 - 10 \times 7 + R_{By} \times 9 = 0$

or, $-62.5 - 70 + R_{By} \times 9 = 0$

$\therefore R_{By} = 14.72 kN$

$(\uparrow^+) \Sigma F_y = 0$

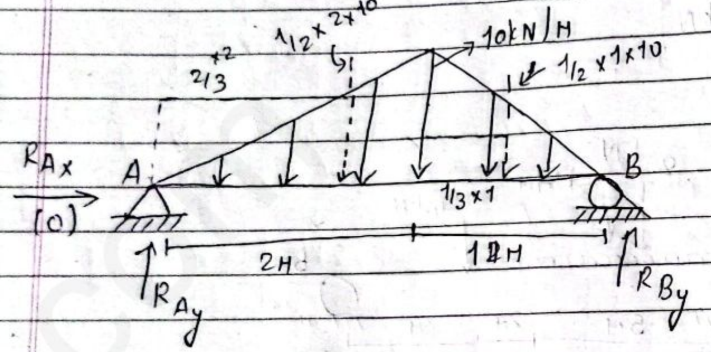
or, $R_{Ay} - 25 - 10 + R_{By} = 0$

$\therefore R_{Ay} = 20.28 kN (\uparrow)$

$(\circlearrowleft^{+})\ \Sigma M_B = 0$

or, $-20.28 \times 9 + 25 \times (2.5 + 4) + 10 \times 2 = 0$

$\therefore\ -0.02 \cong 0$ checked



Sol^n

$(\rightarrow^{+})\ \Sigma F_x = 0$

$R_{Ax} = 0$

$(\circlearrowleft^{+})\ \Sigma M_A = 0$

or, $-\frac{1}{2} \times 2 \times 10 \times \left(\frac{2}{3} \times 2\right) - \frac{1}{2} \times 1 \times 10 \times \left(2 + \frac{1}{3}\right) + R_{By} \times 3 = 0$

or, $-\frac{40}{3} - \frac{35}{3} + R_{By} \times 3 = 0$

$\therefore\ R_{By} = \frac{25}{3} = 8.33$ kN

$(\uparrow^{+})\ \Sigma F_y = 0$

or, $R_{Ay} - \frac{1}{2} \times 2 \times 10 - \frac{1}{2} \times 1 \times 10 + R_{By} = 0$

or, $R_{Ay} = 5 + 10 - 8.33$

$\therefore\ R_{Ay} = 6.67$ kN $(\uparrow)$

$(\circlearrowleft^{+})\ \Sigma M_B = 0$

or, $-6.67 \times 3 + \frac{1}{2} \times 2 \times 10 \times \left[\frac{1}{3} \times 2 + 1\right] + \frac{1}{2} \times 1 \times 10 \left[\frac{2}{3} \times 1\right] = 0$

$\therefore\ -0.01 \cong 0$ (checked)

Tutorial 1 a, c, d, g, h, i, j, k, m, n

Pg: 170   Calculate the reaction force.

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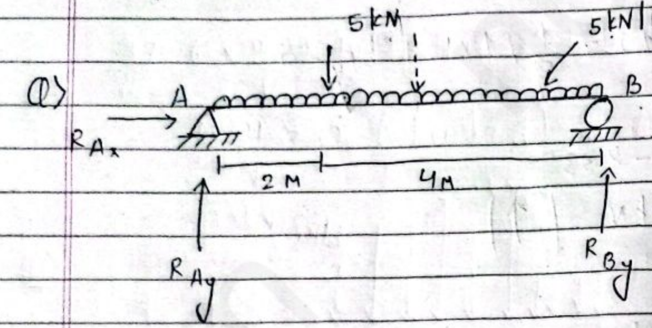
Applied Mechanics Manual (Static)

Q)



Sol^n

$(\xrightarrow{+})\ \Sigma F_x = 0$

$R_{Ax} = 0$

$(\circlearrowleft +)\ M_A = 0$

$-5 \times 2 - 5 \times 6 \times \frac{6}{2} + R_{By} \times 6 = 0$

$\therefore R_{By} = \frac{100}{6} = 16.67\ kN$

$(\uparrow +)\ \Sigma F_y = 0$

$R_{Ay} - 5 - 30 + R_{By} = 0$

or, $R_{Ay} - 35 + 16.67 = 0$

$\therefore R_{Ay} = 18.33\ kN$

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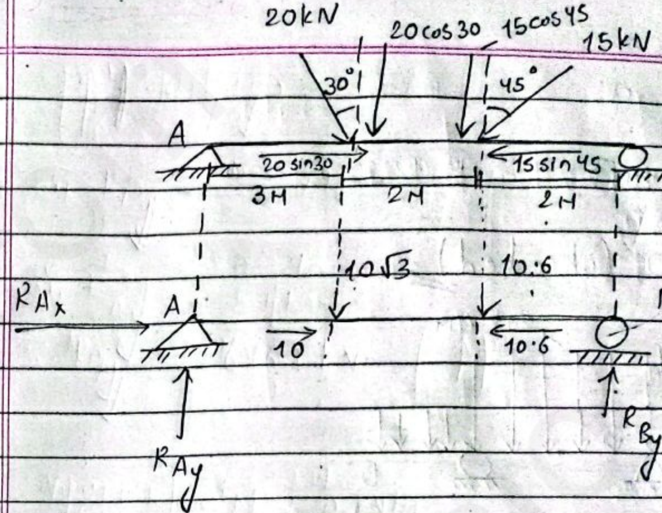
Q.



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $R_{Ax} + 10 - 10.6 = 0$

$\therefore R_{Ax} = 0.6\ kN\ (\rightarrow)$

$(\circlearrowleft +)\ \Sigma M_A = 0$

$-10\sqrt{3} \times 3 - 10.6 \times 5 + R_{By} \times 7 = 0$

or, $-104.96 + R_{By} \times 7 = 0$

$\therefore R_{By} = \frac{104.96}{7} = 15\ kN\ (\uparrow)$

$(\uparrow +)\ \Sigma F_y = 0$

$R_{Ay} - 10\sqrt{3} - 10.6 + R_{By} = 0$

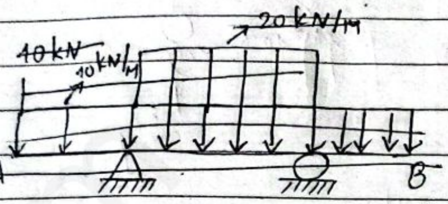
$\therefore R_{Ay} = 12.92\ kN.$

Check

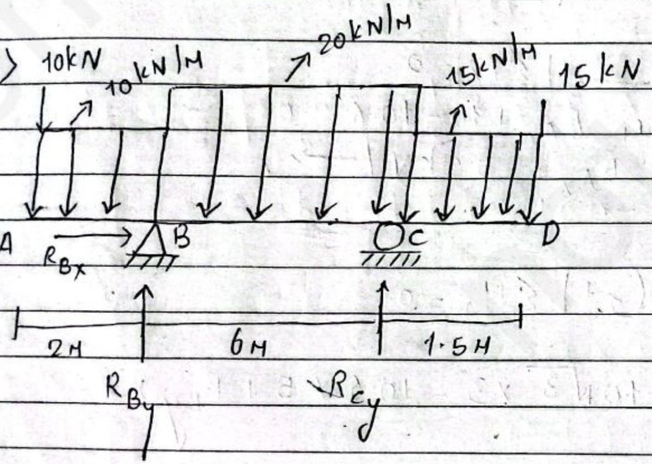
$(\circlearrowright +) M_B = 0$

or, $+10.6 \times 2 + 10\sqrt{3} \times 4 - 12.92 \times 7 = 0$

$\therefore$ $0.04 \simeq 0$ (checked)



Q Tutorial 2>



Sol^n

$(\longrightarrow +) \sum F_x = 0$

$\therefore R_{Bx} = 0$

$(\circlearrowright +) \sum M_B = 0$

$10 \times 2 + 10 \times 2 \times \left(\frac{2}{2}\right) - 20 \times 6 \times \frac{6}{2} + R_{Cy} \times 6 - 15 \times 1.5 \times \left[\frac{1.5}{2} + 6\right]$

$- 15 \times 7.5 = 0.$

or, $20 + 20 - 360 + R_{Cy} \times 6 - 151.875 - 112.5 = 0$

or, $R_{Cy} = \frac{584.375}{6}$

$\therefore R_{Cy} = 97.39$ kN.

$(\uparrow +) \sum F_y = 0$

or, $-10 - 20 - 120 - 15 \times 1.5 - 15 + R_{By} + R_{Cy} = 0.$

$\therefore R_{By} = 90.11$ kN.

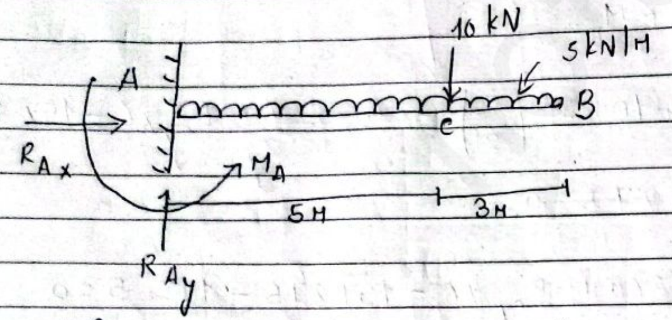
$(\circlearrowright +) \sum M_C = 0$

$10 \times 8 + 10 \times 2 \times \left[\frac{2}{2} + 6\right] - 90.11 \times 6 + 20 \times 6 \times \left(\frac{6}{2}\right)$

$- 15 \times 1.5 \times \left(\frac{1.5}{2}\right) - 15 \times 1.5 = 0$

$\therefore -0.035 \simeq 0$ (check)

Q)



Sol^n

$(\xrightarrow{+}) \sum F_x = 0$

$\therefore R_{A_x} = 0$

$(\circlearrowleft +) \sum M_A = 0$

$+M_A - 10\times5 - 5\times8\times\frac{8}{2}$

$\therefore M_A = 210$ kNm $(\circlearrowleft +)$

$(\uparrow +) \sum F_y = 0$

$R_{A_y} - 10 - 5\times8 = 0$

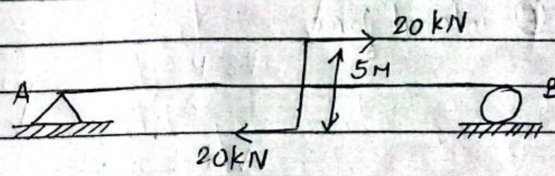
$\therefore R_{A_y} = 50$ kN.

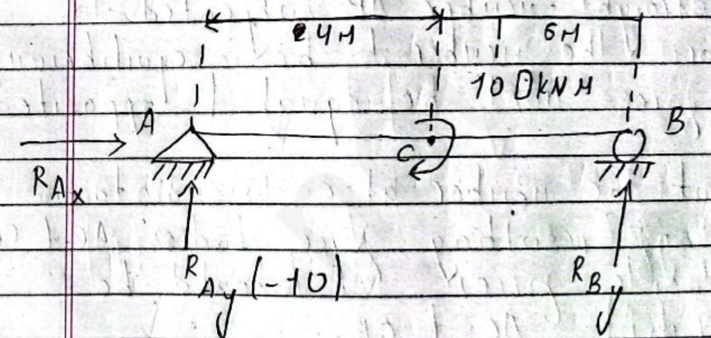
$(\circlearrowleft +) M_B = 0$

or, $M_B\ 210 - 50\times8 + 10\times3 + 5\times8\times\frac{8}{2} = 0$

$\therefore 0 = 0$ (checked).

Q)





$(\xrightarrow{+}) \sum F_x = 0$

$\therefore R_{A_x} = 0$

$(\circlearrowleft +) \sum M_A = 0$

$-100 + R_{B_y}\times10 = 0$

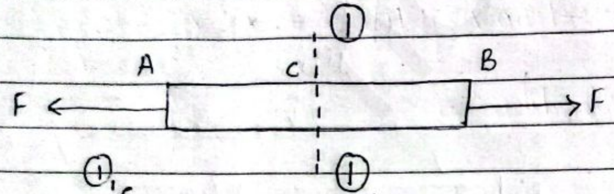
$\therefore R_{B_y} = 10$ kN $(\uparrow)$

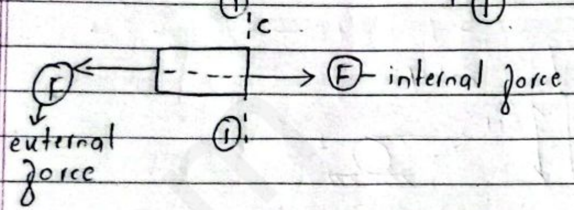
$(\uparrow +) \sum F_y = 0$

$R_{A_y} + 10 = 0$

$\therefore R_{A_y} = -10$ kN $(\downarrow)$

Internal forces in a Beam:

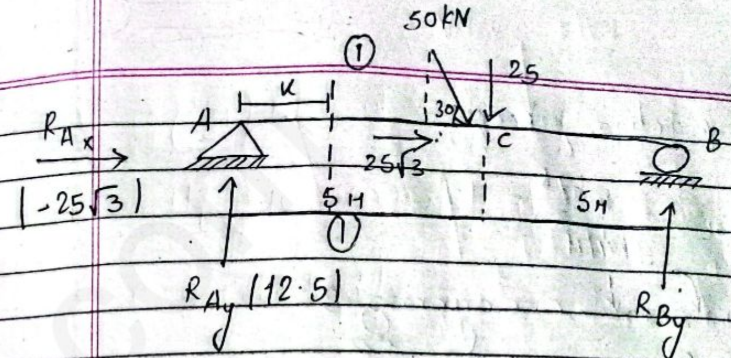




Consider a straight member AB subjected to force F. For the member to be in equilibrium, the two forces must be equal & opposite.

Suppose, we cut the member at C. To maintain equilibrium of resulting force bodies AC & CB, equal & opposite forces F should be applied in AC & CB.

Because the two parts AC & CB were in equilibrium before the member was cut, internal forces equivalent to these forces must have existed in the member itself.



$(\xrightarrow{+})\ \Sigma F_x = 0$

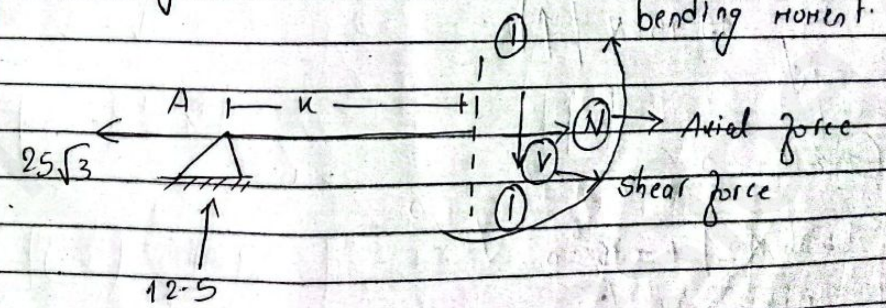
$R_{Ax} + 25\sqrt{3} = 0$

$\therefore R_{Ax} = -25\sqrt{3}\ kN\ (\leftarrow)$

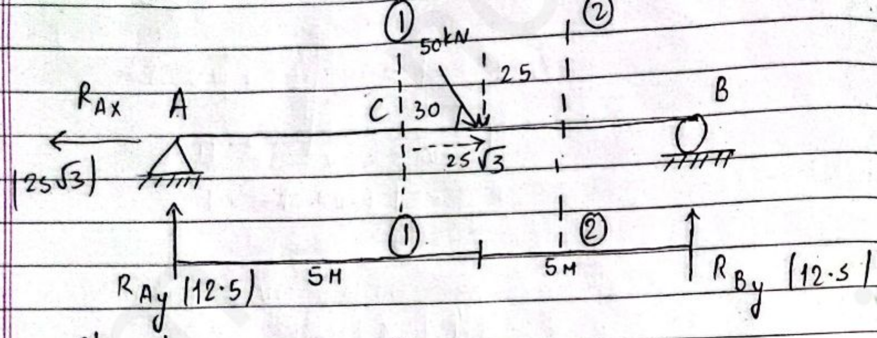
$(\circlearrowleft +)\ \Sigma M_B = 0$

$-R_{Ay} \times 10 + 25 \times 5 = 0$

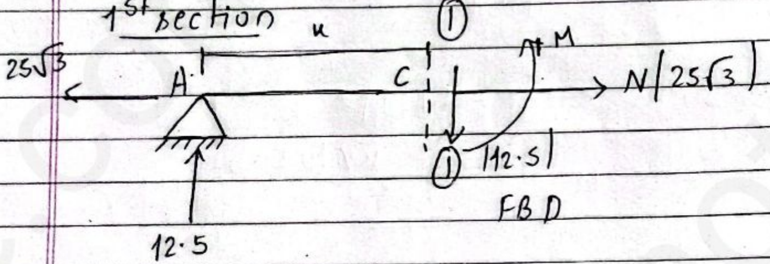
$\therefore R_{Ay} = 12.5\ kN\ (\uparrow)$



Internal force contd.....



1st section



$(\xrightarrow{+})\ \Sigma F_x = 0$

$-25\sqrt{3} + N = 0$

$\therefore N = 25\sqrt{3}$ kN $(\longrightarrow)$

$(\uparrow^{+})\ \Sigma F_y = 0$

$12.5 - V = 0$

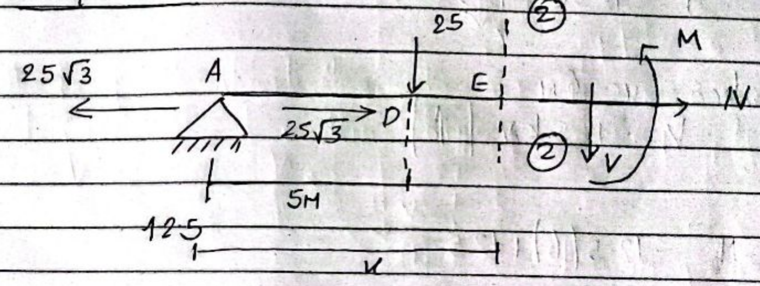
$\therefore V = 12.5$ kN $(\downarrow)$

$(\circlearrowleft^{+})\ \Sigma M_A = 0$

$-12.5x + M = 0$

$\therefore M = 12.5x$

2nd section :



$(\xrightarrow{+})\ \Sigma F_x = 0$

$-25\sqrt{3} + 25\sqrt{3} + N = 0$

$\therefore N = 0$

$(\uparrow^{+})\ \Sigma F_y = 0$

$12.5 - 25 - V = 0$

$V = -12.5$ kN $(\uparrow)$

$(\circlearrowright +)\ \Sigma M_E = 0$

$-12.5x + 25(x-5) + M = 0$

$M = 12.5x - 25(x-5)$

$\therefore M = -12.5x + 125$

when $x = 6m$

$V = -12.5kN\ (\uparrow)$

$M = -12.5(6) + 125$

$= 50kNm$.

Axial force (N)

At any cross section of a member, it is the algebraic sum of all forces acting parallel to the longitudinal axis on either side of the section.

Shear force (V).

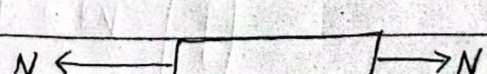
At any cross section of a member, it is the algebraic sum of all the forces acting transverse to the longitudinal axis of the member on either side of the section.

Bending Moment (M)

At any cross section of a member, it is the algebraic sum of the moments taken about an axis passing through the centroid of the section on either side of section.

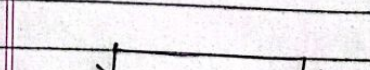
Sign convention:

1) Axial force

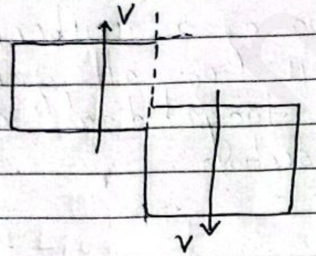


Tensile force (+ve)

Compressive force (-ve)



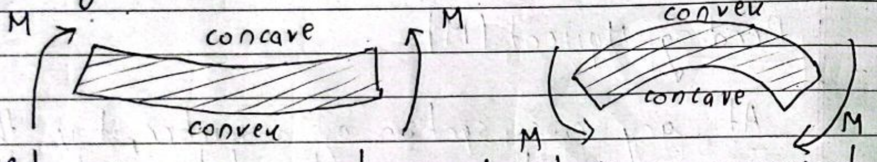
2) Shear force.



(+ve)

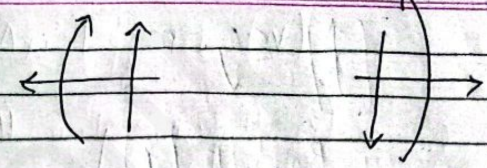
Shear force is +ve when it tends to push the left portion upward with r.t the right.

3) Bending moment



(+ve) Sagging moment (-ve) Hogging moment

Bending moment is +ve when it tends to bend the beam concave upwards.

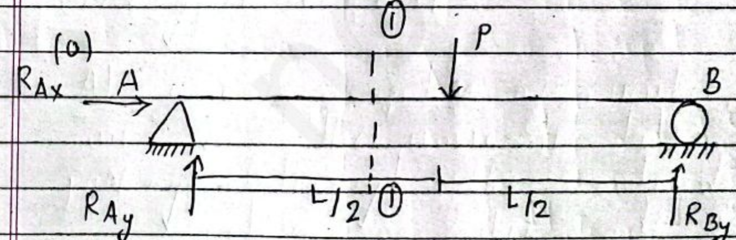


Sign convention of beam.

Axial force diagram, shear force diagram & bending moment diagram:

The graphical representation of the axial force, shear force and bending moment which are introduced in the structural member are called AFD, SFD & BMD.

(a)



Find axial force, shear force and bending moment at $x = L/4$ and $x = 2L/3$.

Soln

$(\xrightarrow{+}) \sum F_x = 0$

$\therefore R_{Ax} = 0$

$(\circlearrowright +)\ \Sigma M_A = 0$

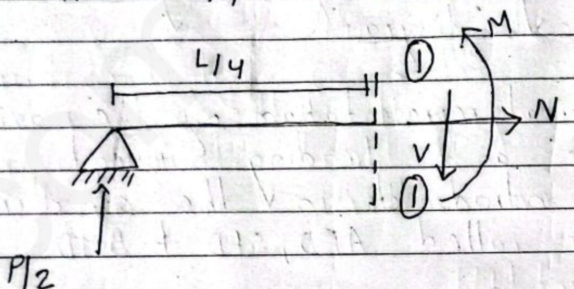
$-P \cdot \frac{L}{2} + R_{By} \cdot L = 0$

$\therefore R_{By} = P/2\ (\uparrow)$

$(\uparrow)\ \Sigma F_y = 0$

$R_{Ay} = \frac{P}{2}\ (\uparrow)$

at $x = L/4$.

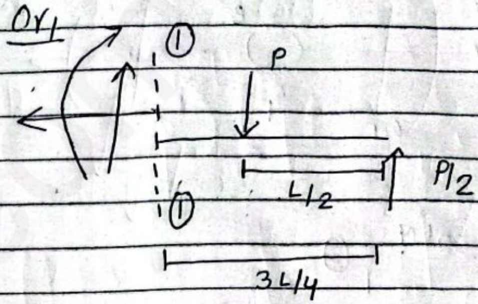


$N = 0$

$V = \frac{P}{2}$

$M = \frac{P}{2} \cdot \frac{L}{4} = \frac{PL}{8}$

Or,



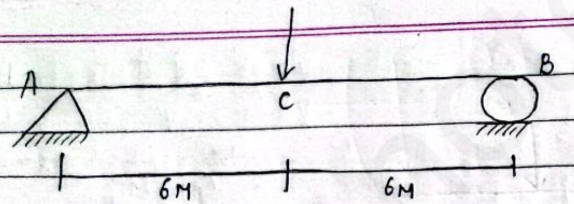
$N = 0$

$V = -P/2 + P = P/2$

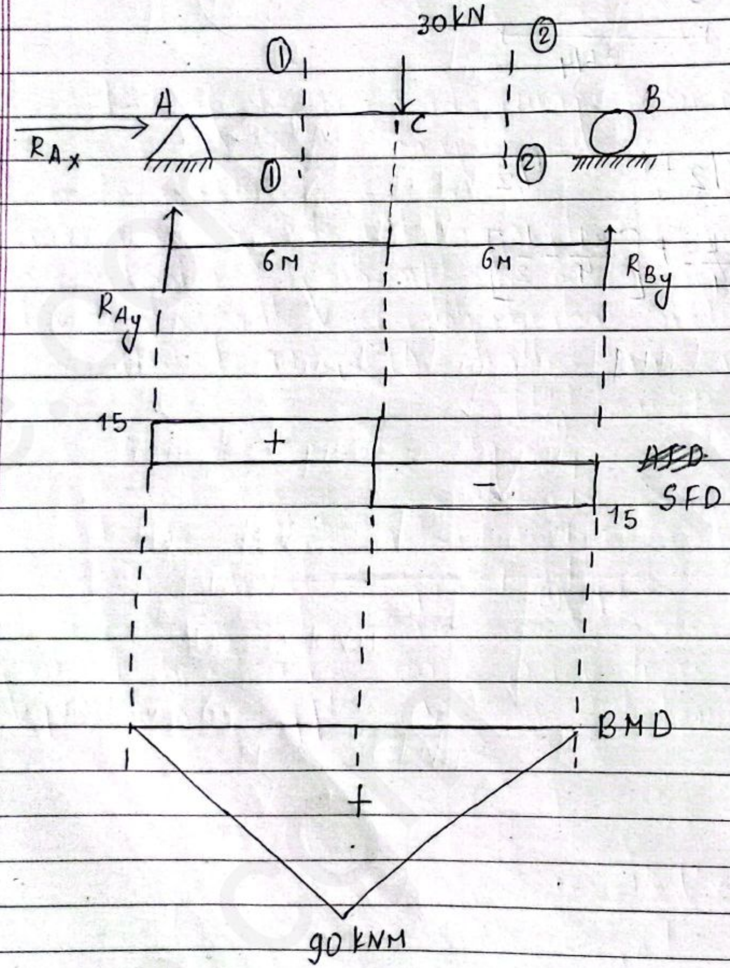
$M = \frac{P}{2} \cdot \frac{3L}{4} - P\left[\frac{3L}{4} - \frac{L}{2}\right]$

$= \frac{PL}{8}$

• Draw AFD, SFD, BMD



From F.B.D,



Sol^n Calculation of reactions,

$(\xrightarrow{+})\ \Sigma F_x = 0$

$R_{Ax} = 0\ (\rightarrow)$

$(\uparrow +)\ \Sigma F_y = 0$

$R_{Ay} - 30 + 15 = 0$

$\therefore R_{Ay} = 15\,kN\ (\uparrow)$

$(\circlearrowright +)\ \Sigma M_A = 0$

or, $-30 \times 6 + R_{By} \times 12 = 0$

$\therefore R_{By} = 15\,kN\ (\uparrow)$

To check,

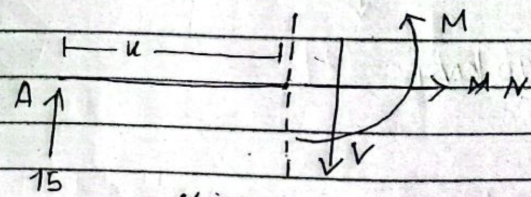
$(\circlearrowright +)\ \Sigma M_B = 0$

or, $-15 \times 12 + 30 \times 6 = 0$

or, $-180 + 18 = 0$

$\therefore 0 = 0$ (checked).

Axial Force Diagram (AFD),



$N = 0$

$V_x = 15\,kN$

Shear force diagram (SFD)

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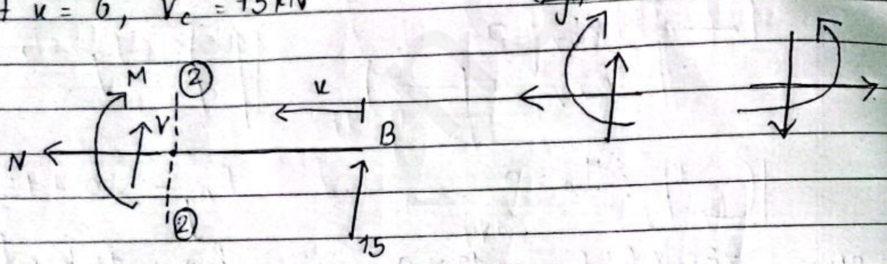
At $x=0$, $V_A = 15$ kN
At $x = 6$, $V_c = 15$ kN

Sign convⁿ:



$V_x = -15$ kN
at $x = 0$, $V_B = -15$ kN
at $x = 6$, $V_c = -15$ kN

Bending moment diagram (BMD)

From the previous figure with section ①,

$M_x = 15x$ (linear variation)
At $x = 0$,
$M_A = 0$ kNm
At $x = 6$,
$M_c = 90$ kNm

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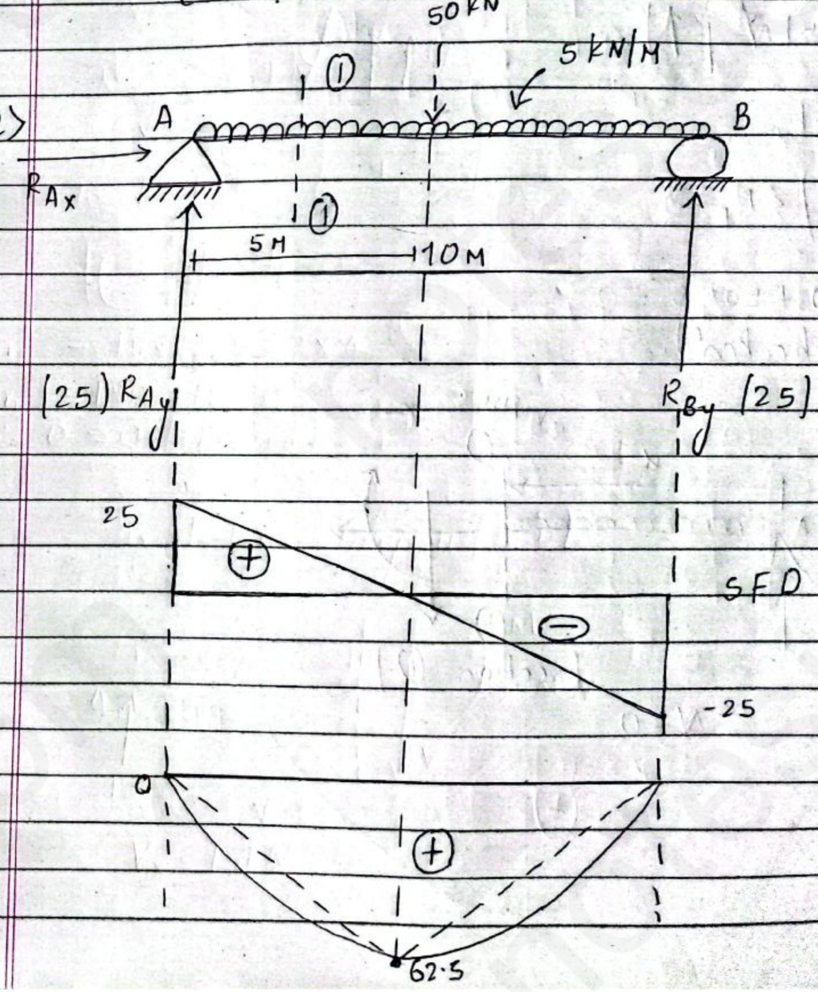
From the previous figure with section ②,

$M_x = 15x$ (linear variation)
At $x = 0$,
$M_B = 0$
At $x = 6$,
$M_c = 90$ kNm

2)



Calculation of reactions,

$(\xrightarrow{+}) \sum F_x = 0$

$\therefore R_{A_x} = 0$

$(\circlearrowleft +) \sum M_A = 0.$

or, $-50 \times 5 + R_{B_y} \times 10 = 0$

$\therefore R_{B_y} = 25\,kN \;(\uparrow)$

$(\uparrow +) \sum F_y = 0$

or, $R_{A_y} - 50 + 25 = 0$

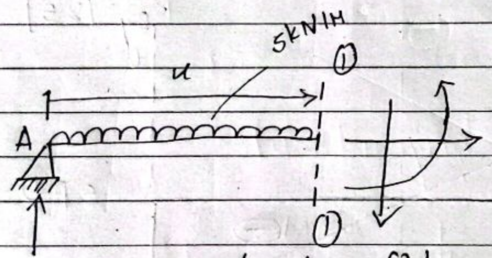
$\therefore R_{A_y} = 25\,kN \;(\uparrow)$

To check,

$(\circlearrowleft +) \sum M_B = 0$

or, $-25 \times 10 + 50 \times 5 = 0$

$\therefore 0 = 0$ (checked).



(section (1))

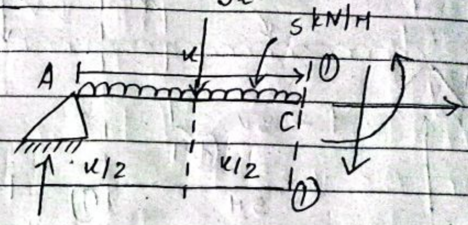
N = 0

Axial Force Diagram

$N_x = 0$. (from section (1) figure).

Shear Force Diagram

$V_{x} = ?$



$V_x = 25 - 5u$ (linear variation).

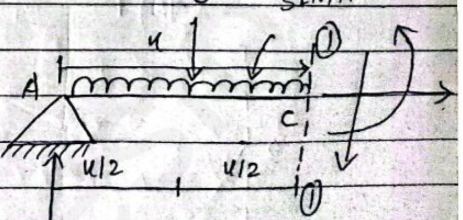
when $u = 0$,

$V_A = 25\,kN$.

when $u = 10$,

$V_B = -25\,kN$

Bending Moment Diagram,



$M_x = 25u - 5u \cdot \frac{u}{2}$

$= 25u - \frac{5u^2}{2}$

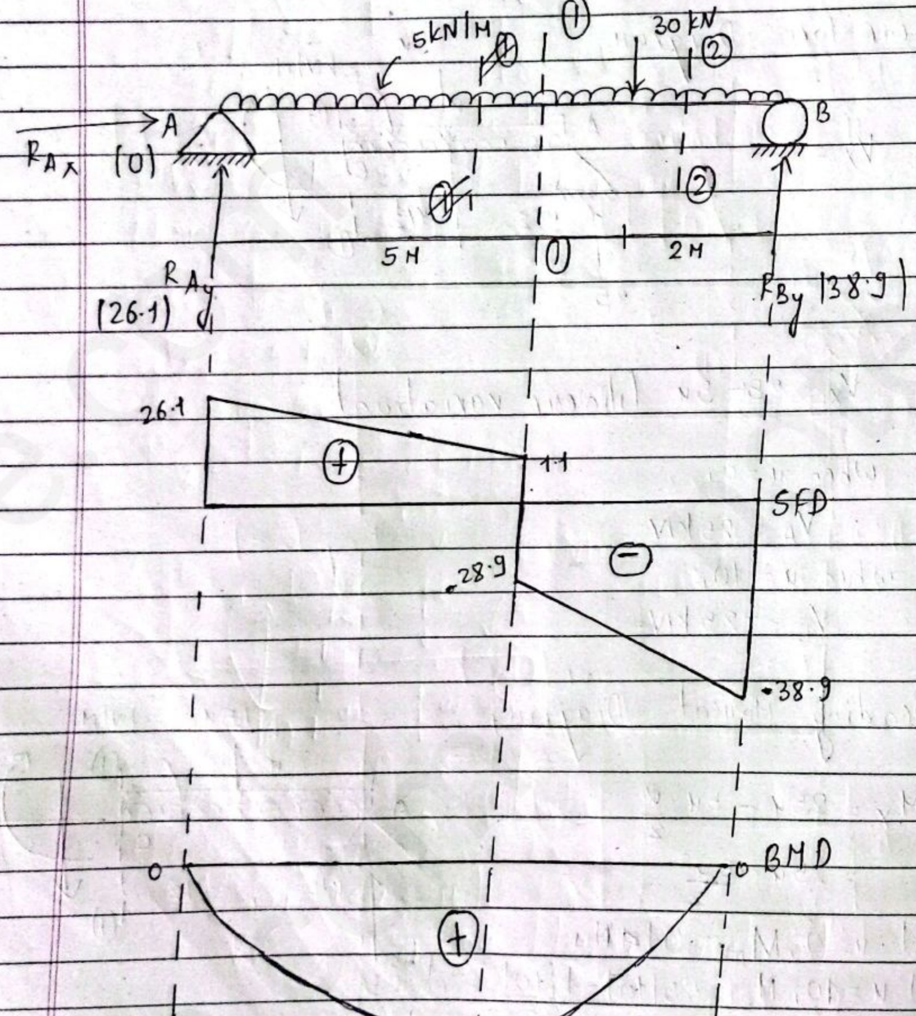
At $u = 0$, $M_A = 0\,kNm$.

At $u = 10$, $M_B = 25(10) - \frac{5(10)^2}{2} = 0\,kNm$ 25

At $u = 5$, $M_C = 25(5) - \frac{5(5)^2}{2} = 62.5\,kNm$

at $x = 2.5\,m$, $M = 25(2.5) - \frac{5}{2}(2.5)^2 = 46.8\ kNm.$

at $x = 7.5\,m$, $M = 25(7.5) - \frac{5}{2}(7.5)^2 = 46.8\ kNm.$



Calculation of reactions,

$(\xrightarrow{+})\ \Sigma F_x = 0$

$R_{Ax} = 0.$

$(\uparrow +)\ \Sigma F_y = 0$

$R_{Ay} - 5 \times 7 + 38.9 - 30 = 0$

$\therefore R_{Ay} = 26.1\ kN\ (\uparrow)$

$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $-5 \times 7 \times \frac{7}{2}\ 30 \times 5 + R_{By} \times 7 = 0$

$\therefore R_{By} = 38.9\ kN\ (\uparrow)$

To check,

$(\circlearrowleft +)\ \Sigma M_B = 0$

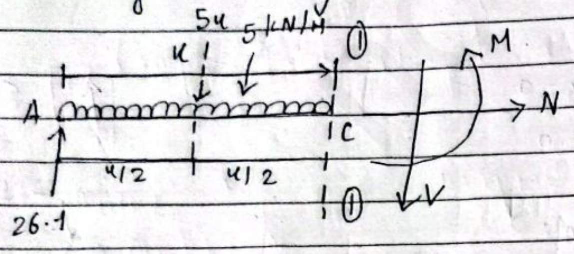
or, $-26.1 \times 7 + 35 \times \frac{7}{2} + 30 \times 2 = 0$

$\therefore -0.2 \approx 0$ (checked).

Axial force Diagram,

$N = 0$

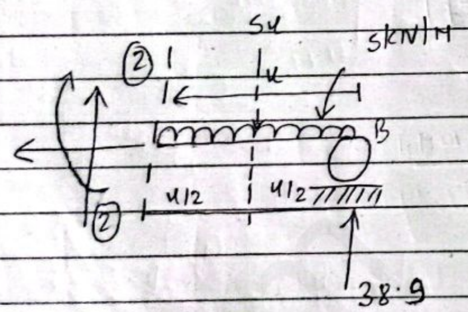
Shear force Diagram (SFD).



26.1

$V_x = 26.1 - 5u$ (linear variation)
(for $0 \le u \le 5M$),

when $u = 0$,
$V_A = 26.1$ kN
when $u = 5$,
~~$V_C = 26.1 - 25 = 1.1$ kN~~



38.9

~~V~~ for $0 < u < 2M$)
$V_x = -38.9 + 5u$ (linear variation)

when $u = 0$,

$V_B = -38.9$ kN

when $u = 2$,

$V = -28.9$ kN.

Bending Moment Diagram (BMD).

From previous figure,
for $0 \le u \le 5$
$M_x = 26.1u - 5u \cdot \frac{u}{2} = 26.1u - \frac{5u^2}{2}$

when $u = 0$,
$M_A = 0$
when $u = 5$,
$M_C = 26.1(5) - \frac{5(5)^2}{2} = 68$ kNM.

when $u = 5/2$,
$M = 26.1\left(\frac{5}{2}\right) - \frac{5}{2}\left(\frac{5}{2}\right)^2 = 49.63$ kNM.

for $0 \le u \le 2M$,
$M_x = 38.9u - 5u \cdot \frac{u}{2}$

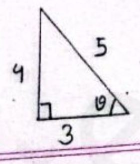
when $u = 0$,
$M_B = 0$

when $u = 1$,
$M = 36.4$ kNM

when $u = 2$,
~~$M_C = 67.8$ kNM.~~

$\tan\theta = \frac{4}{3}$

$\Rightarrow \theta = \tan^{-1}(4/3)$



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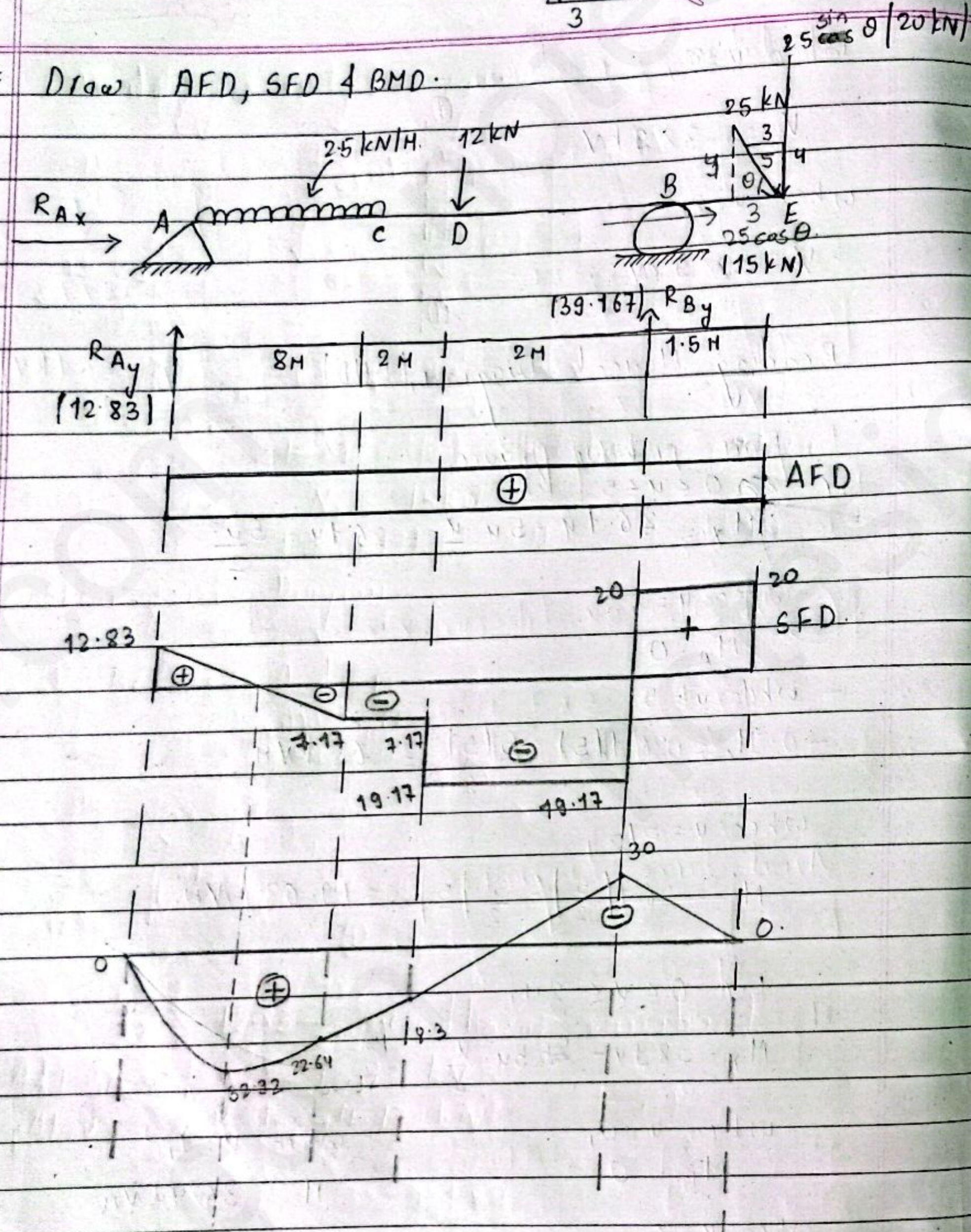
Draw AFD, SFD & BMD.



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Sol^n

Computation of reactions

$(\xrightarrow{+}) \ \Sigma F_x = 0$

$R_{Ax} + 15 = 0$

$\therefore R_{Ax} = -15 \text{ kN} \ (\leftarrow)$

$(\circlearrowleft +) \ \Sigma M_A = 0$

or, $-80 - 12 \times 10 + R_{By} \times 12 - 20 \times 13.5 = 0$

$\therefore R_{By} = 479.5/12 = 39.167 \ (\uparrow)$

$(\uparrow +) \ \Sigma F_y = 0$

$R_{Ay} - 2.5 \times 8 - 12 + 39.17 - 20 = 0$

$\therefore R_{Ay} = 12.83 \text{ kN} \ (\uparrow)$

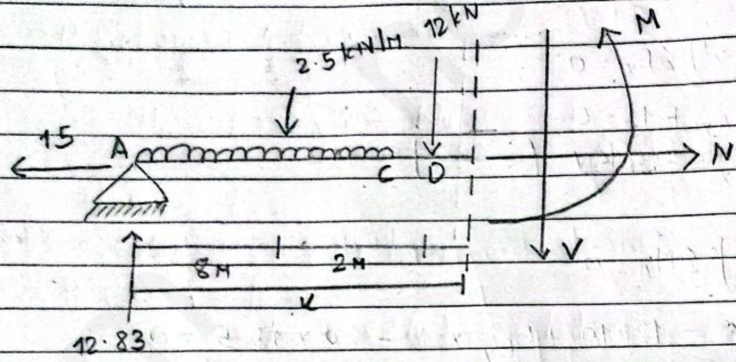
To check,

$(\circlearrowleft +) \ \Sigma M_B = 0$

$-12.83 \times 12 + 2.5 \times 8 \times [4 + 4] + 12 \times 2 - 20 \times 1.5 = 0.$

$\therefore 0.04 \simeq 0$ (checked)

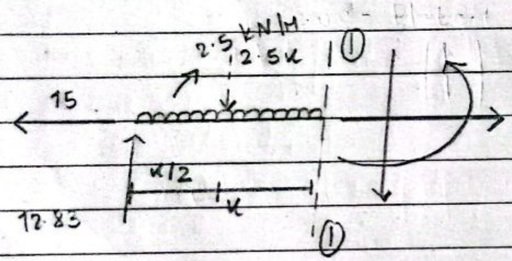
Axial force Diagram



$N_x = 15 kN$

Shear force Diagram

① Between A & C,

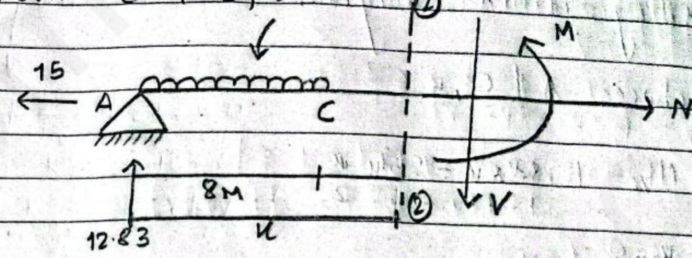


$V_x = 12.83 - 2.5x$ (linear variation)

At $x = 0$, $V_A = 12.83$ kN

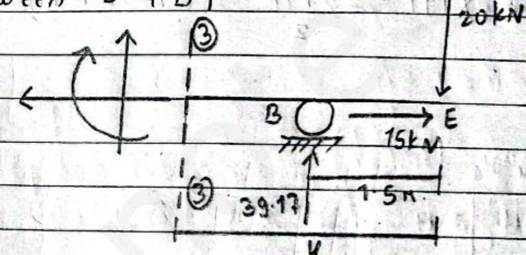
At $x = 8$, $V_C = -7.17$ kN

② Between C & D, 2.5kN/M



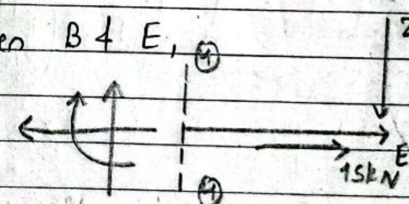
$V_x = 12.83 - 2.5 \times 8 = -7.17 kN$ (constant)

③ Between D & B,



$V_x = 20 - 39.17 = -19.17 kN$ (constant)

④ Between B & E,



$V_x = 20 kN$ constant.

Bending moment diagram,

1) Between A & C,

$M_x = 12.83x - 25x \cdot \frac{x}{2}$

At $x = 0$,

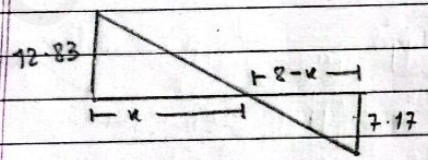
$M_A = 0$

At $x = 8$,

$M_C = 12.83(8) - \frac{25(8)^2}{2} = 22.64$ kNm.

At $x = 4$m,

$M_{mid} = 12.83 \times (4) - \frac{25(4)^2}{2} = 31.32$ kNm



By similar Δ,

$\frac{12.83}{x} = \frac{7.17}{8-x}$

$\Rightarrow x = 5.13$m.

At $x = 5.13$m,

$M_{Max} = 12.83(5.13) - \frac{25(5.13)^2}{2}$

$= 32.92$ kNm

Again, for concave or convex को lagi,

At $x = \frac{5.13}{2}$,

$M = 12.83 \times \frac{5.13}{2} - 25 \times \frac{\left(\frac{5.13}{2}\right)^2}{2}$

$= 22.64$ kNm

Again, at $x = 2.565$, $M = 24.68$ kNm.

2) Between C & D,

for $8 \le x \le 10$m

$M_x = 12.83x - 2.5 \times 8 \times [x - 4]$

$M_x = -7.17x + 80$ // linear variation.

At $x = 8$, $M_C = 22.64$ kNm

At $x = 10$m, $M_D = 8.3$ kNm.

3> Between B & D,

for $1.5 \le x \le 3.5m$

$M_x = -20x + 39.17(x-1.5)$

$= 19.17x - 58.75$ (linear)

At $x = 3.5m$,

$M_B = 8.34$ kNm.

At $x = 1.5$

$M = -30$ kNm

4> Between B & E,

for $0 \le x \le 1.5m$.

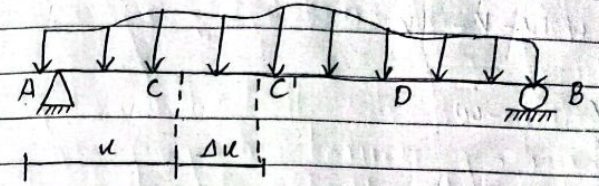
$M_x = -20x$ (linear)

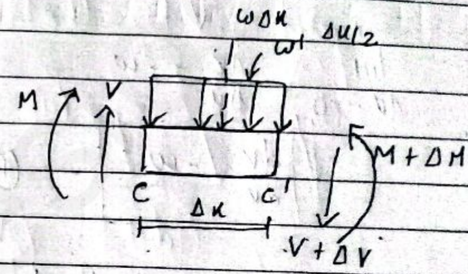
For, $x = 0$, $M_E = 0$

For $x = 1.5$, $M_B = -30$ kNm

IMP

Relation between load, shear and bending moment.





Let us consider a simply supported beam AB carryin a distributed load w per unit length and c let C & C' be two points of the beam at a distance Δx from each other. The shear and bending moment at C will be denoted by V & M respectively. The shear & bending moment at C' will be denoted by V+ΔV & M+ΔM respectively.

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Page

$(\uparrow^{+}) \sum F_y = 0$

$V - w\Delta x - (V + \Delta V) = 0$

$V - w\Delta x - V - \Delta V = 0$

$\frac{\Delta V}{\Delta x} = -w$

$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -w$

$$\boxed{\frac{dV}{dx} = -w} \quad \text{--- (I)}$$

This equation indicates that for a beam loaded as shown in figure, the slope $\frac{dV}{dx}$ of the shear curve is negative, the numerical value of the slope at any point is equal to the load per unit length at that point.

Integrating eq^n (I) bet^n. C & D,

$\int_C^D dV = \int_C^D -w\,dx$

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$V_D - V_C = -$ Area of load curve between C & D - (II)

eq^n (I) & (II) cease to be valid when concentrated loads are applied bet^n C & D since they do not take into account the sudden change in shear caused by concentrated load. Hence, these equations should be applied ~~when~~ between successive concentrated loads only.

Relation between shear & bending moment:

$(\circlearrowleft^{+}) \sum M_C = 0$

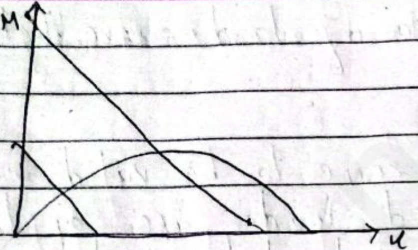
$-M - V\Delta x + w\,\Delta x \cdot \frac{\Delta x}{2} + M + dM = 0$

$\Delta M = V\Delta x - \frac{w(\Delta x)^2}{2}$

Dividing both sides by Δx;

$\lim_{x \to 0} \frac{\Delta M}{\Delta x} = V - \frac{w}{2} \cdot \Delta x$

$$\boxed{\frac{dM}{dx} = V}$$



eqⁿ (III) indicates that the slope $\frac{dM}{dx}$ of the bending moment diagram is equal to the value of shear

Integrating eqⁿ (III) betⁿ C & D

$$\int_C^D dM = \int_C^D V\,dx$$

$M_D - M_C$ = Area of shear curve betⁿ C & D.

$\frac{dM}{dx} = 0$

This implies that bending moment is maximum when shear force is zero.

Important properties of SFD & BMD.

1> The ordinates of the SFD change suddenly at point of application of point loads and there is sudden change in the slope of BMD at these locations

2> Ordinates of SFD remain constant between two points while BMD will be oblique straight line.

3> Under UDL, SFD varies linearly but BMD varies parabolically.

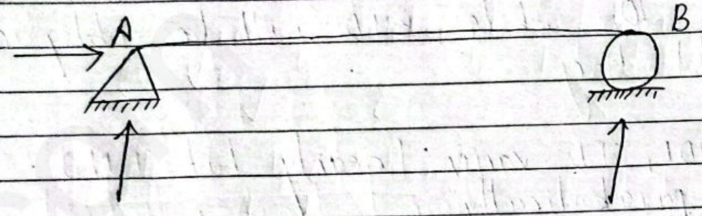
4> BM is zero at simple supports and hinges.

5> At ~~simple~~ support, the shear is equal to reaction.

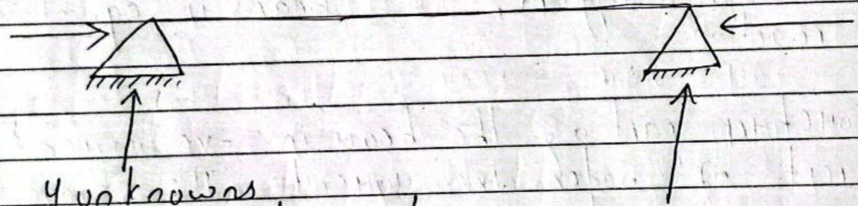
6> Sometimes part of the beam is -ve moment and rest of under +ve moment. The change from -ve to +ve occurs through a point of zero moment. This point is called point of inflection or contraflexure. The beam deflects to opposite nature of curvature on either side of point.

7) Bending moment is maximum at point where shear force is zero.

Determinacy and indeterminacy of structural system.



3 unknowns
3 independent equation ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$)



4 unknowns
3 independent equations.

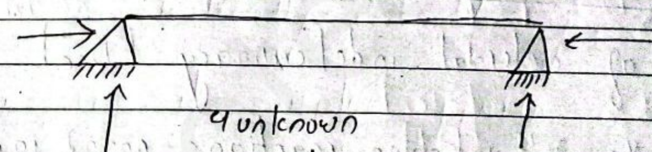
Statically determinate structure:

→ Any structure whose reaction components or internal stresses can be calculated by using equations of static equilibrium alone.

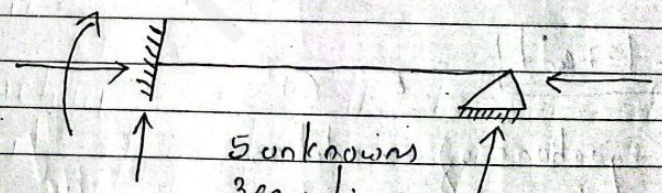
Statically indeterminate structure:

→ ... cannot be calculated...

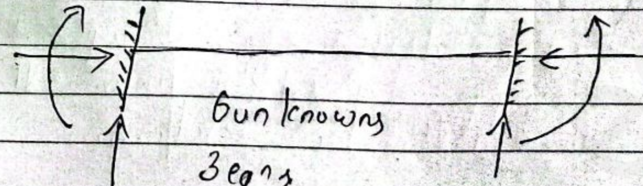
Degrees of static indeterminacy (D_s):



4 unknown
3 equations



5 unknowns
3 equations.



6 unknowns
3 eqns

→ The no. of additional equations necessary for the addition solution of statically indeterminate structures is known as degree of static indeterminacy of structure.

Degree of static indeterminacy of a structure is sum of

(I) Degree of external indeterminacy (D_{se})

(II) Degree of internal indeterminacy (D_{si})

i.e. $D_s = D_{se} + D_{si}$

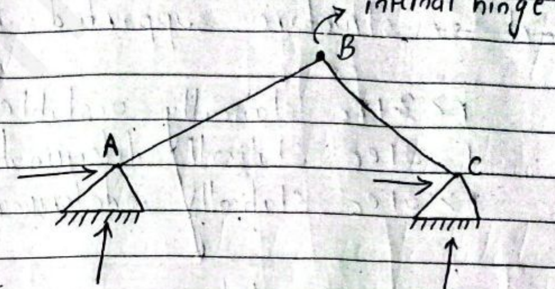
Degree of external indeterminacy (D_{se}):

D_{se} = No. of unknown reactions − no. of independent equilibrium equations.

$$D_{se} = r - (3 + e_c)$$

e_c = equations of condition

Equation of condition:



$r = 4$

$D_{se} = r - (3 + e_c)$

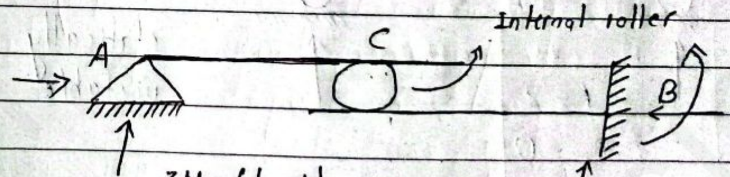
$= 4 - 4$

$= 0$

$\Sigma M_{B_{left}} = 0$ or, $\Sigma M_{B_{right}} = 0$

Now, $e_c = 1$

So, $D_{se} = 4 - (3+1) = 0$



$\Sigma M_c(left) = 0$ or $\Sigma M_c(right) = 0$

$\Sigma F_{x_c}(left) = 0$ or $\Sigma F_{x_c}(right) = 0$

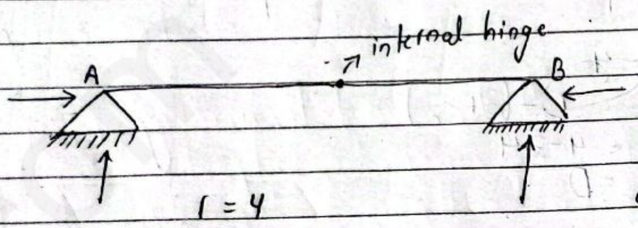
$e_c = 2$

So, $D_{se} = 5 - (3+2) = 0$

Summary:

→ For a ~~support so~~ structure supported by 'r' reactions

$r < 3 + e_c$ statically unstable.
$r = 3 + e_c$ statically determinate externally.
$r > 3 + e_c$ statically indeterminate externally

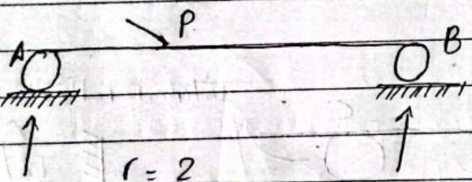


Statically determinate

$r = 4$

$e_c = 1$

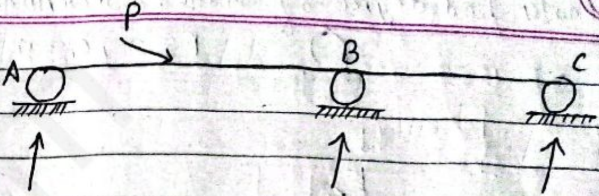
$D_{se} = r - (3 + e_c)$
$= 4 - (3+1)$
$= 0$



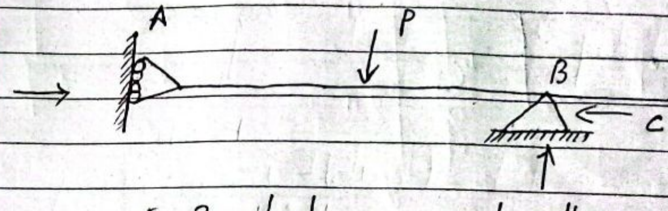
statically unstable

$r = 2$

$e_c = 0$



geometrically unstable.



$r = 3$ but, geometrically unstable

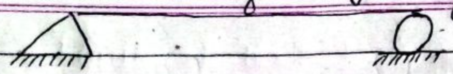
If force are either parallel or concurrent, then it is geometrically unstable.

A support may be supported by a sufficient number of reactions but still may be unstable due to improper arrangement of supports. Such structures are referred as geometrically unstable structure.
For a structure to be geometrically stable, it must be supported by at least three reactions all of which must be neither parallel nor concurrent.

HW

Determine if the structure is stable or not. Also determine if it is statically determinate or indeterminate. If indeterminate find degree of static indeterminacy.

i) $r = 3$, $ec = 0$



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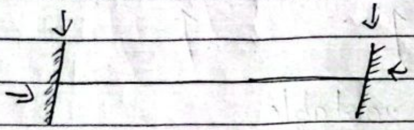

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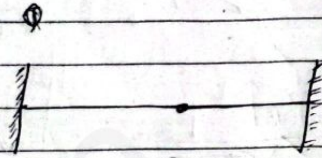
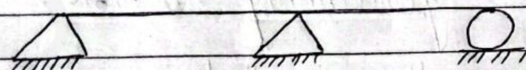
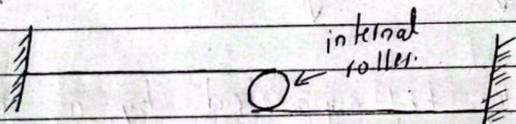


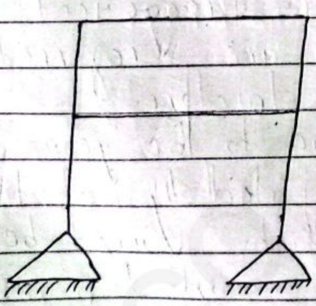
iv)



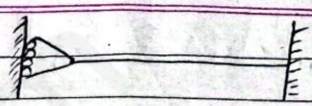
v)



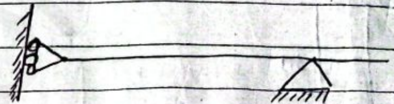
vi)



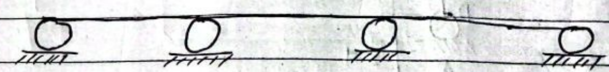
vii)



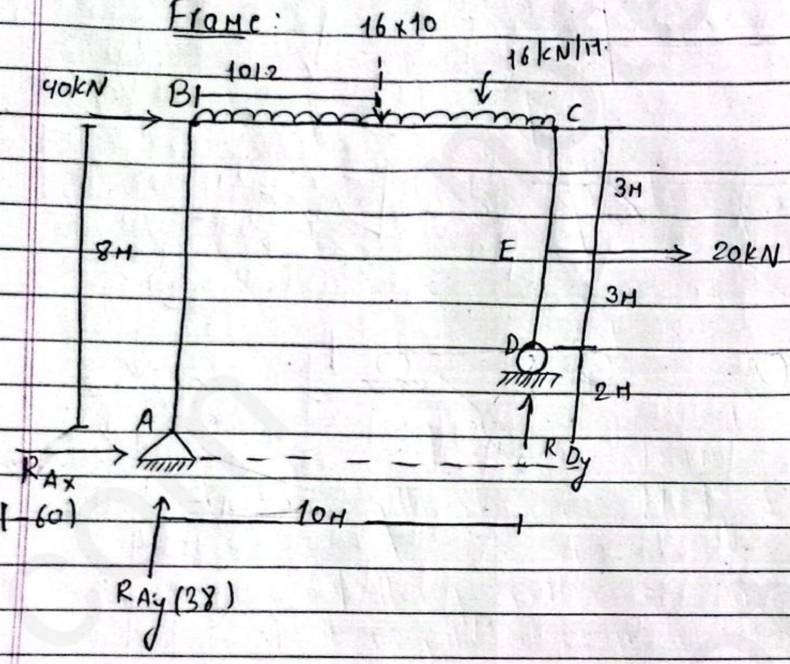
viii)

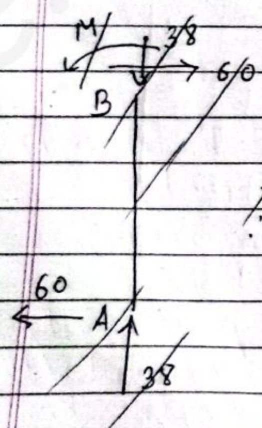


ix)



Frame:





$(\circlearrowleft +)\ \Sigma M_B = 0$

$-60 \times 8 + M = 0$

$\therefore M = 60 \times 8 = 480\ kNm.$

PTO

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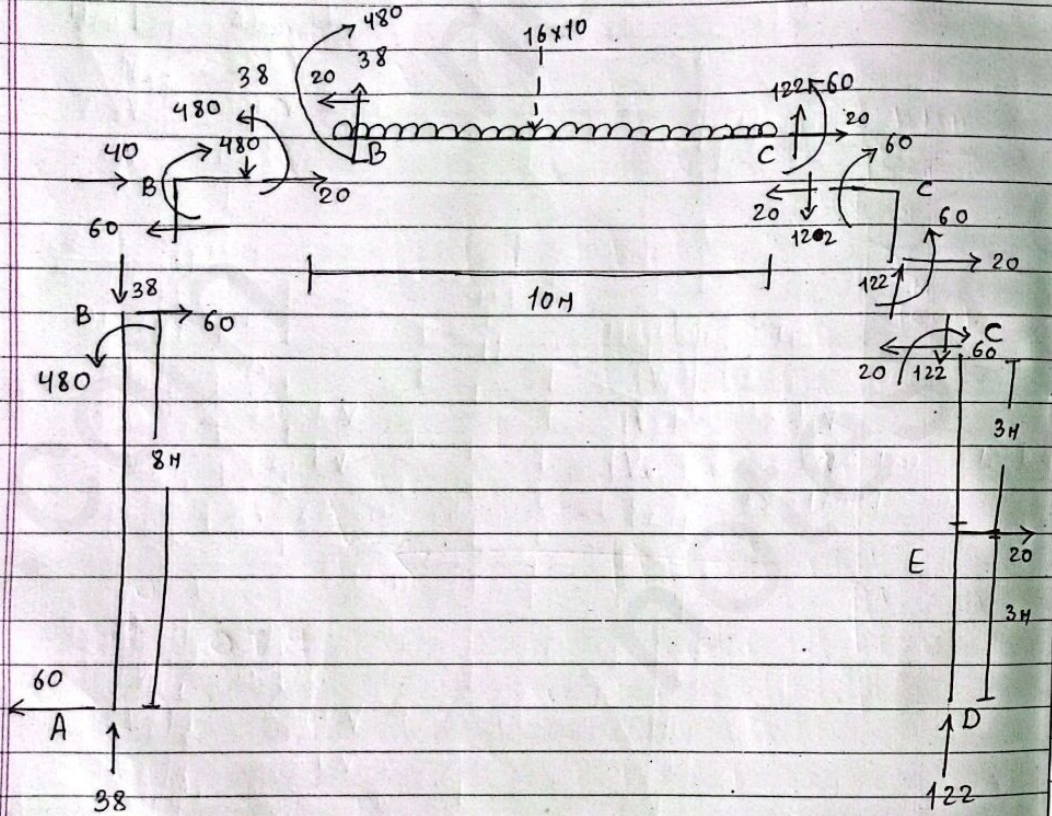

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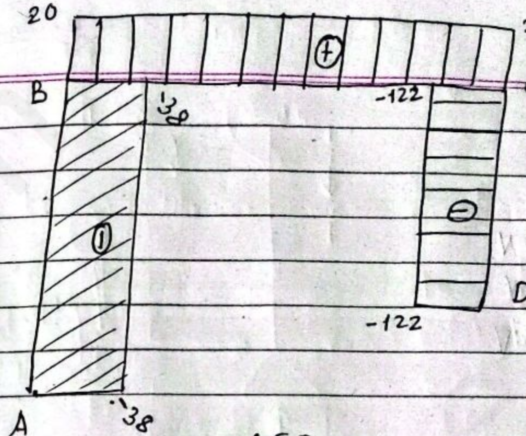
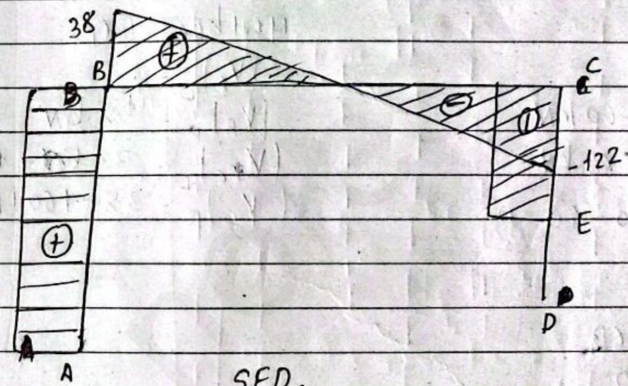
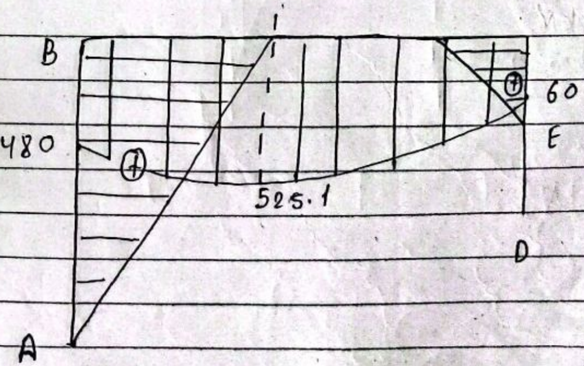


AFD



SFD.



BMD

Date ____ Page ____

Axial force,

Member AB

$N_{AB} = -38\,kN$

$N_{BC} = 20\,kN$

$N_{CD} = -122\,kN$

Shear force,

Member AB

$(V_A)_L = 0$

$(V_A)_R = +60\,kN$

$(V_B)_L = +60\,kN$

$(V_B)_R = 60 - 60 = 0$

Member BC

$(V_B)_L = 0$

$(V_B)_R = 38\,kN$

$(V_C)_L = 38 - 160 = -122\,kN$

$(V_C)_R = 38 - 160 + 122 = 0$

Member CD

$(V_C)_L = 0$

$(V_C)_R = -20\,kN$

$(V_E)_L = -20\,kN$

$(V_E)_R = -20 + 20 = 0$

$V_D = 0$

Date ____ Page ____

Bending Moment,

Member AB

$M_A = 0$

$(M_B) = 60 \times 8 = +480\,kNm$

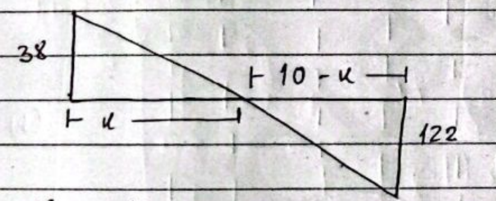
Member BC

$M_B = 480\,kNm$

$M_C = 60\,kNm$

$M_{2.375} = 480 + 38 \times 2.375 - 16 \times 2.375 \times \frac{2.375}{2}$

$= 525.125\,kNm$



$\frac{38}{x} = \frac{122}{10 - x}$

$x = 2.375\,m$

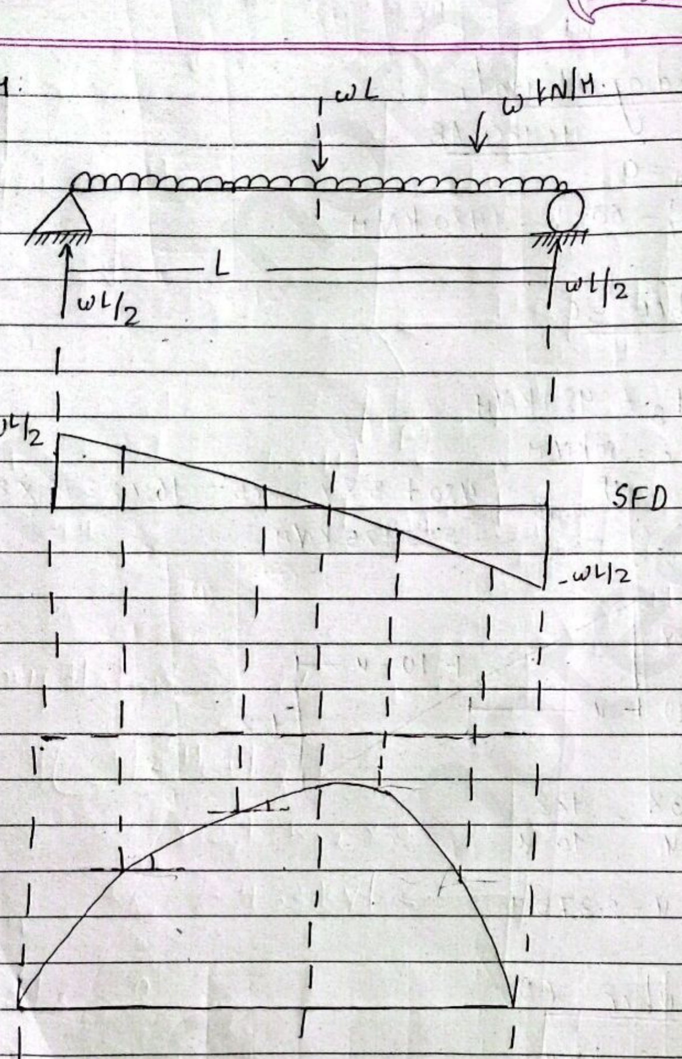
Member CD

$M_C = 60\,kNm$

$M_E = 60 - 20 \times 3 = 0$

$M_D = 60 - 20 \times 6 + 20 \times 3 = 0$

Beam:



Shear force

$(V_A)_L = 0$

$(V_A)_R = \frac{wL}{2}$

$(V_B)_L = \frac{wL}{2} - wL = -\frac{wL}{2}$

$(V_B)_R = -\frac{wL}{2} + \frac{wL}{2} = 0$

Bending Moment,

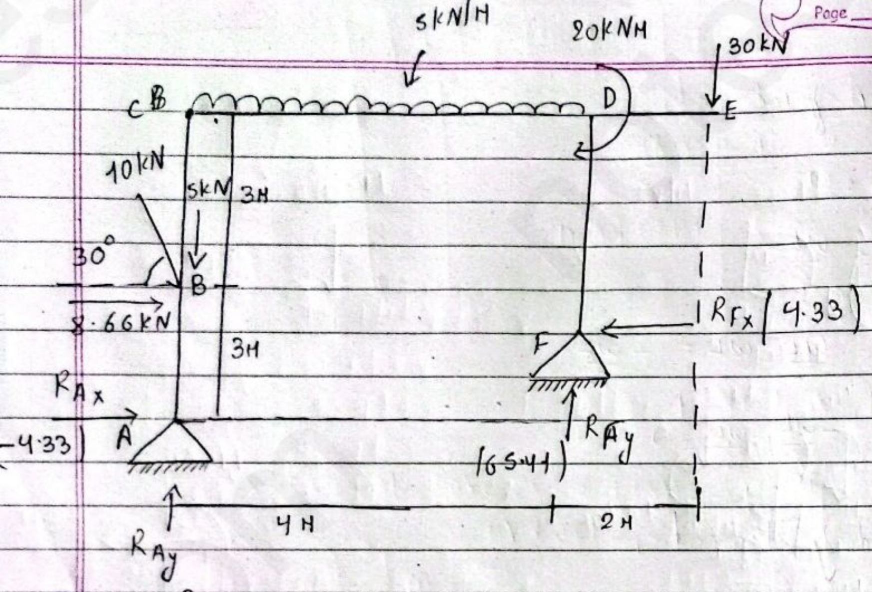
$M_A = 0$

$M_B = 0$

$M_{Max} = \frac{wL}{2} \cdot \frac{L}{2} - \frac{wL}{2} \left(\frac{L}{2} \cdot \frac{1}{2} \right)$

$= \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}$

[13]



Soln

computation of R:

$(\circlearrowright +) \Sigma M_c(\text{left}) = 0$

$8.66 \times 3 + R_{Ax} \times 6 = 0$

$R_{Ax} = -4.33\ kN\ (\leftarrow)$

$(\rightarrow +) \Sigma F_x = 0$

$-4.33 + 8.66 - R_{Fx} = 0$

$\therefore R_{Fx} = 4.33\ kN\ (\leftarrow)$

$(\circlearrowright +) \Sigma M_c(\text{right}) = 0$

$-5 \times 4 \times \frac{4}{2} - 20 - 30 \times 6 - 4.33 \times 5 + R_{Fy} \times 4 = 0$

$\therefore R_{Fy} = 65.41\ kN\ (\uparrow)$

$(\uparrow +) \Sigma F_y = 0$

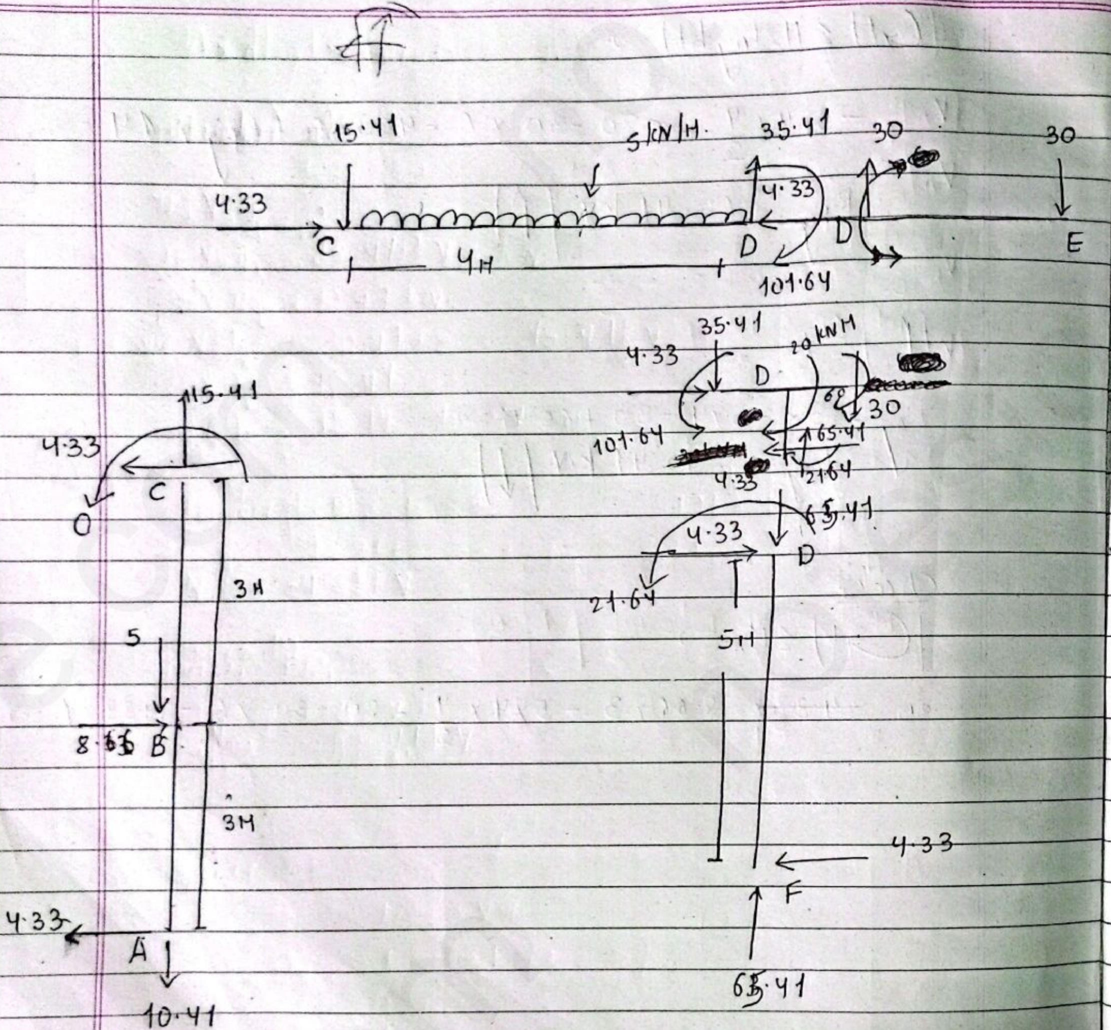
$R_{Ay} - 5 - 5 \times 4 - 30 + 65.41 = 0$

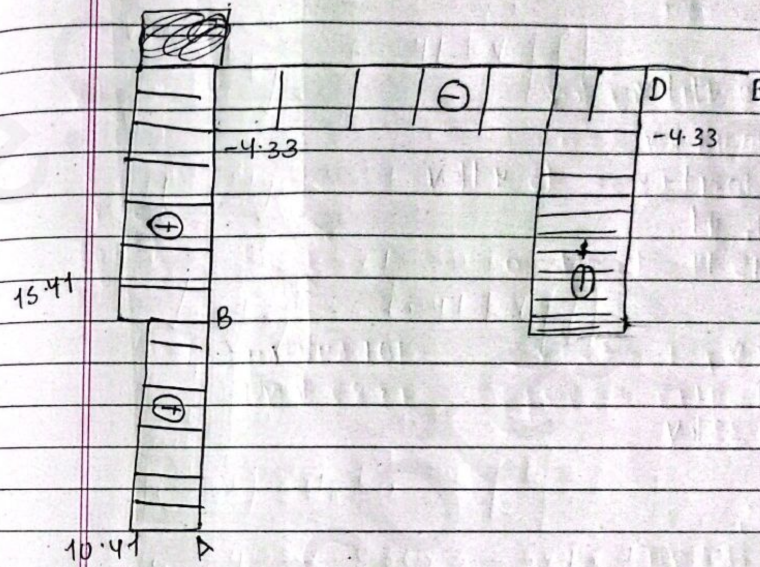
$\therefore R_{Ay} = -10.41\ kN\ (\downarrow)$

Check,

$(\circlearrowright +) \Sigma M_A = 0$

or, ~~$4.33 \times$~~ $8.66 \times 3 - 5 \times 4 \times \frac{4}{2} - 20 - 30 \times 6 - 4.33 \times 1 = 0$


15.41
5 kN/M
35.41
30
30
4.33
C
D
D
E
4M
4.33
101.64
35.41
20 kNM
4.33
D
30
101.64
65.41
21.64
4.33
15.41
4.33
C
3M
5
8.66
B
3M
4.33
A
10.41
4.33
D
65.41
21.64
5M
4.33
F
65.41


D
E
-4.33
-4.33
15.41
B
10.41
A

Axial Force,

Member AC,

$(N_A)_L = 0$
$(N_A)_R = 10.41$ kN
$(N_B)_L = 10.41$ kN
$(N_B)_R = 10.41 + 5 = 15.41$ kN.
$(N_C)_L = 15.41$
$(N_C)_R = 15.14 - 15.41 = 0.$

Member CD,

$N_{CD} = -4.33$ kN.

Member DF,

$N_{DF} = -65.41$ kN.

Shear force,

Member AB,

$(V_A)_L = 0$
$(V_A)_R = 4.33$
$(V_B)_L = 4.33$
$(V_B)_R = -4.33$
$(V_C)_L = -4.33$
$(V_C)_R = 0.$

Member DE,

$(V_D)_L = 0$
$(V_D)_R = 30$
$(V_E)_L = 30$
$(V_E)_R = 30 - 30 = 0$

Member DF,

$(V_D)_L = 0$
$(V_D)_R = 4.33$
$(V_F)_L = 4.33$
$(V_F)_R = 4.33 - 4.33$
$= 0.$

Bending moment,

Member AC,

$M_A = 0$

$M_B = 4.33 \times 3 = 12.99$ kN.

$M_C = 4.33 \times 6 - 8.66 \times 3 = 0$.

Member CD,

$M_C = 0$.

$M_D = -15.41 \times 4 - 20 \times 2 = -101.64$ kNM

$M_{mid} = -15.41 \times 2 - 5 \times 2 \times \frac{2}{2} = -40.82$ kNM.

Member DE,

$M_D = -60$ kNM.

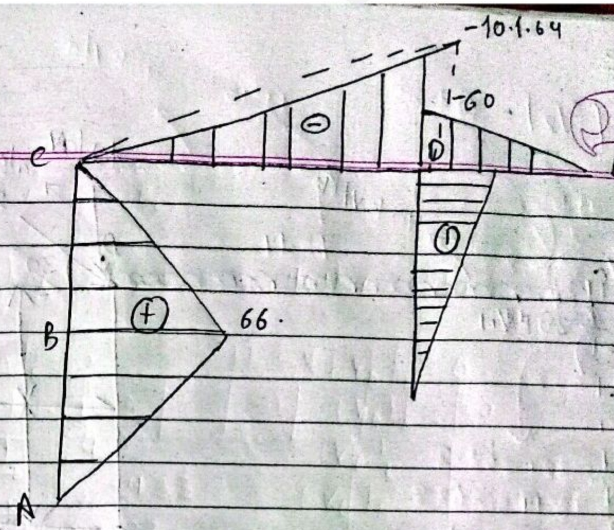
$M_E = -60 + 30 \times 2 = 0$.

Member DF.

$M_D = -21.65$ kNM

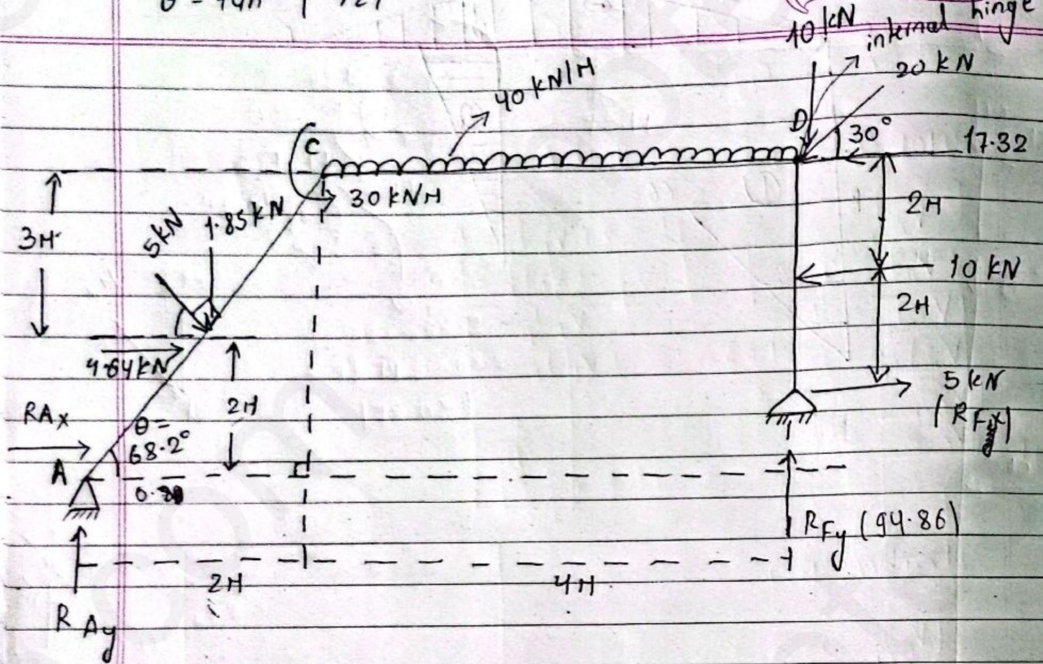
$M_F = -21.65 + 4.33 \times 5$

$= 0$.



$\tan\theta = \frac{5}{2}$

$\theta = \tan^{-1}(5/2) = 68.2°$



Soln

Computation of rxn:

$(\circlearrowleft +)\ \Sigma M_{D\,right} = 0$

$-10 \times 2 - R_{Fx} \times 4 = 0$

$\therefore R_{Fx} = -5\ kN\ (\longrightarrow)$

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $R_{Ax} + 4.64 - 17.32 - 10 + 5 = 0$

$\therefore R_{Ax} = 17.68\ kN\ (\uparrow)$

$(\circlearrowleft +)\ \Sigma M_D(left) = 0$

$17.68 \times 5 - R_{Ay} \times 6 + 4.64 \times 3 + 1.85 \times [1.2 + 4] + 30 + 40 \times 4 \times \left(\frac{4}{2}\right)$

$\therefore R_{Ay} = 76.99\ kN \approx 77\ kN$

$(\uparrow +)\ \Sigma F_y = 0$

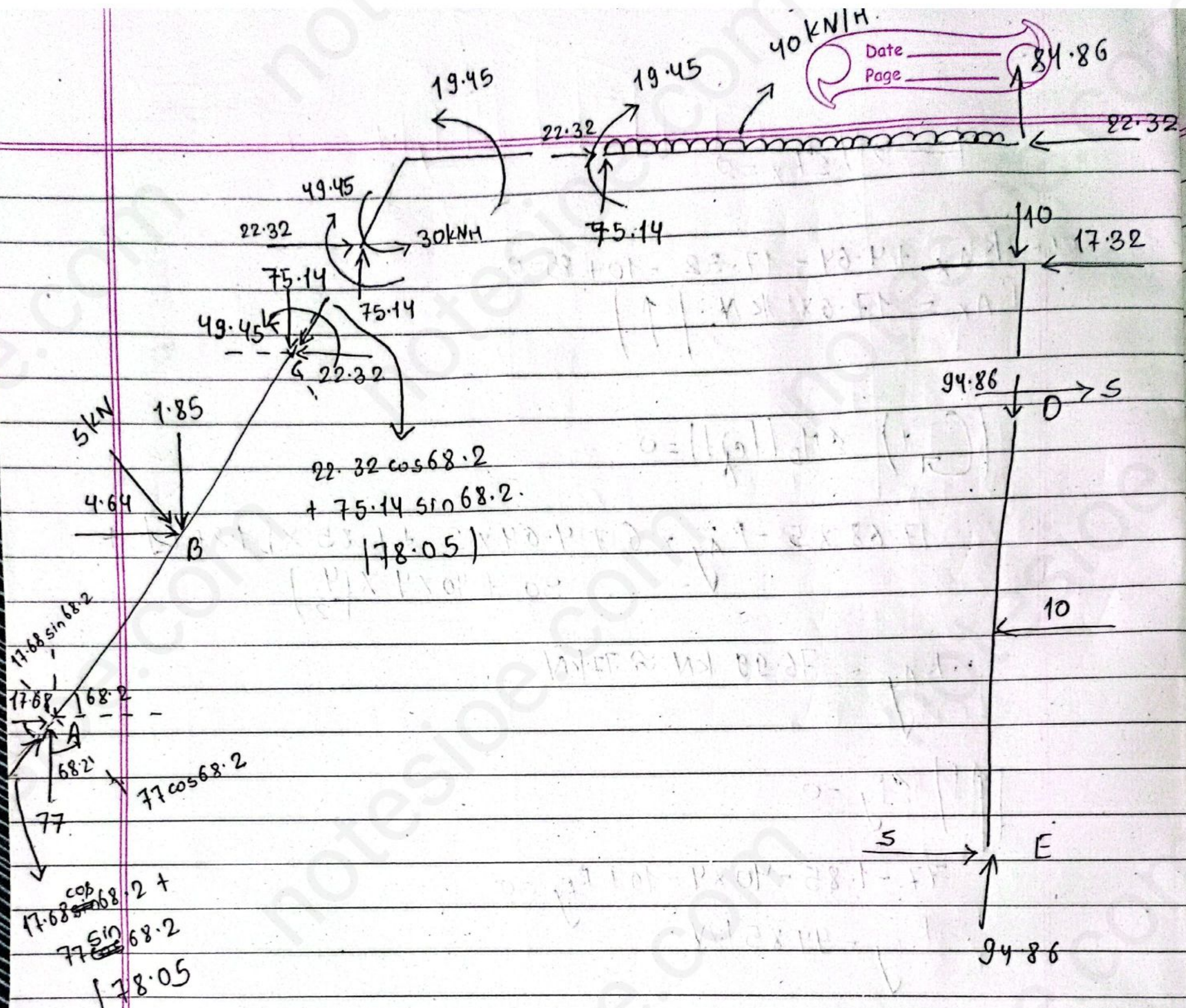
$77 - 1.85 - 40 \times 4 - 10 + R_{Fy} = 0$

$\therefore R_{Fy} = 94.85\ kN.$

To check,

$\Sigma M_A = 0$

or,



do it yourself ☺ HeHe!

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