CHAP-2

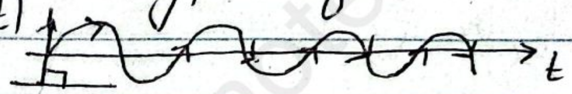
Wave Motion

Wave motion is a form of disturbance which travels through a medium due to the repetitive periodic motion of the particles of the medium about their mean position.

Types of wave:

① On the basis of mode of vibration of the particle. There are two types of waves:

a) Transverse wave



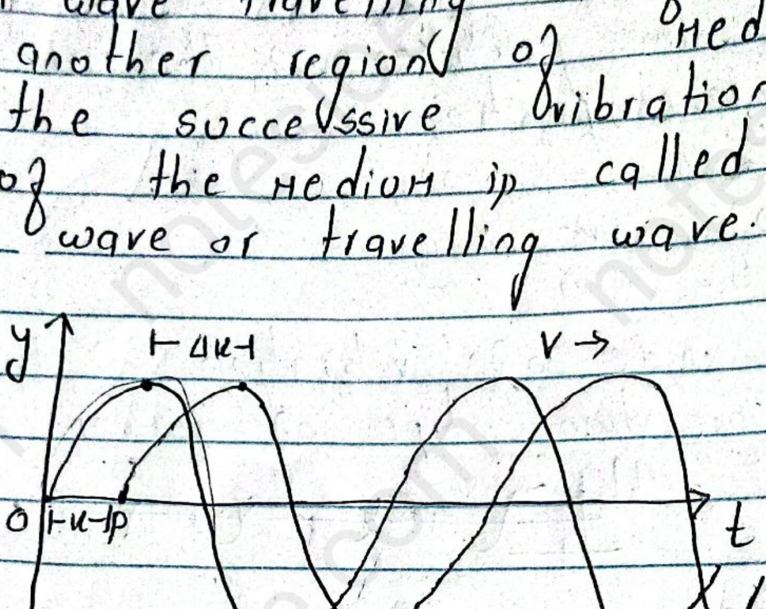
b) Longitudinal wave

a) when the particle of the medium vibrates about their mean position in a direction perpendicular to the direction of propagation of wave. Then the wave is called transverse wave. eg: water wave

b) If the particles of the medium propagating the wave motion vibrate in the direction of propagation, then the wave is called longitudinal wave. eg: sound wave

Equation of propagation of wave (travelling wave):

A wave travelling ~~war~~ from one region to another region of medium through the successive vibration of particle of the medium is called progressive wave or travelling wave.



Suppose a wave originated from 'O', the displacement of the particle at that instant is $y = a\sin\omega t$ — (i)

Also, ~~cond~~ consider another particle 'P' at a distance 'x' from O to its right side.

Let the wave travelling from left to right with velocity 'v' then, displacement of particle at point 'P' will be,

$$y = a\sin(\omega t - \phi) \quad - \text{(ii)}$$

Phase difference,

$$\phi = \frac{2\pi}{\lambda} \times x \quad \text{and also, } \omega = \frac{2\pi}{T}$$

but, $v = \frac{\lambda}{T}$

$$\therefore T = \frac{\lambda}{v}$$

Now, eqⁿ (11) becomes,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

above eqⁿ can be expressed in terms of wave number 'k' $= \frac{2\pi}{\lambda}$ and angular freq 'ω':

$$y = a\sin(\omega t - kx)$$

and ~~from~~ for right to left,

$$\boxed{y = a\sin(\omega t + kx)}$$

IMP.

- Differential eq^n of wave motion:

we have, general wave eqn travelling from left to right in,

$$y = a\sin\frac{2\pi}{\lambda}(vt-x) \quad -①$$

diff eq^n ① w.r.t time, we get,

$$\frac{dy}{dt} = \frac{d\left\{a\sin\left[\frac{2\pi}{\lambda}(vt-x)\right]\right\}}{dt}$$

$$\frac{dy}{dt} = \frac{2a\pi v}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) \quad -(II)$$

diff eq^n ① w.r.t 'x',

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda}a\cos\frac{2\pi}{\lambda}(vt-x) \quad -(III)$$

diff eq^n (II) w.r.t 't', we get

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2}{\lambda^2}v^2 a\sin\frac{2\pi}{\lambda}(vt-x) \quad -(IV)$$

diff eq^n (III) w.r.t 'x',

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2}a\sin\frac{2\pi}{\lambda}(vt-x) \quad -(V)$$

Now, from eqⁿ (II) & (III),

$$\boxed{\frac{dy}{dt} = -v\frac{dy}{dx}}$$

Therefore, velocity of particles = -ve of v.of wave and product of slope of curve.

Also, from eqⁿ (IV) & (V),

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$

Therefore, acclrⁿ of particle at a point is equal to square of wave velocity x curvature of the displacement curve at that point.

Wave velocity & partical velocity:

we have, v is the velocity of wave and 'y' is the depth of particle,

ie. $y = a \sin \frac{2\pi}{\lambda} (vt - x)$.

$\therefore v_{particle} = \frac{dy}{dt}$

$= \frac{2\pi}{\lambda} va \cos \frac{2\pi}{\lambda} (vt - x)$

$(v_{particle})_{Max} = \frac{2\pi}{\lambda} va$

$= \omega a$

And, acceleration of particle,

$a_p = \frac{d^2y}{dt^2}$

$= -\frac{4\pi^2 v^2 a}{\lambda^2} a \left[\sin \frac{2\pi}{\lambda} (vt - x) \right]$

$\therefore (a_p)_{max} = -\frac{4\pi^2 v^2}{\lambda^2} a$

$= -\omega^2 a$

Now,

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda} a\cos^2\frac{\pi}{\lambda}(vt-x)$$

$$\boxed{\therefore V_{particle} = -v\frac{dy}{dx}}$$

Energy, power & intensity of progressive wave.

Total energy is the sum of K.E & PE. Therefore, P.E of the progressive wave is,

$$U = \frac{1}{2}ky^2$$

where, y is the displacement of particle.

we have, $y = a\sin\frac{2\pi}{\lambda}(vt-x)$

substituting ~~Subtracting~~ value of y in $U = \frac{1}{2}ky^2$

$$U = \frac{1}{2}ka^2\sin^2\frac{2\pi}{\lambda}(vt-x)$$

$$= \frac{1}{2}M\omega^2a^2\sin^2\frac{2\pi}{\lambda}(vt-x)$$

Now,

potential energy per unit volume is,

$$U = \frac{1}{2}\rho\omega^2a^2\sin^2\frac{2\pi}{\lambda}(vt-x)$$

And, KE per unit volume is,

$$\frac{1}{2}\rho v^2$$

$$= \frac{1}{2}\rho\left(\frac{dy}{dt}\right)^2$$

$$= \frac{1}{2}\rho\,\frac{4\pi^2 v^2}{\lambda^2} a^2 \cos^2\frac{2\pi}{\lambda}(vt-x)$$

$$= \frac{1}{2}\rho\,\omega^2 a^2 \cos^2\frac{2\pi}{\lambda}(vt-x)$$

$$= \cancel{\frac{1}{2}\rho\,\omega^2 a^2 \cos^2 2}$$

Now,

T.E per unit volume is,

$$E = KE + U$$

$$= \frac{1}{2}\rho\omega^2 a^2\left[\sin^2\frac{2\pi}{\lambda}(vt-x) + \cos^2\frac{2\pi}{\lambda}(vt-x)\right]$$

$$E = \frac{1}{2}\rho\omega^2 a^2$$

Since,

$$v = \lambda f$$

$$E = 2\pi^2\rho f^2 a^2$$

Now, total energy for volume V is,

$$E = 2\pi^2 \rho f^2 a^2 V$$

$$\sqrt{\frac{F}{A\rho}}$$

Since,

$$V = Al = Avt$$

$$\therefore E = 2\pi^2 \rho f^2 a^2 A v t$$

$$E = \frac{1}{2}\rho\omega^2 a^2$$

$$2\pi^2 \rho f^2 a^2$$

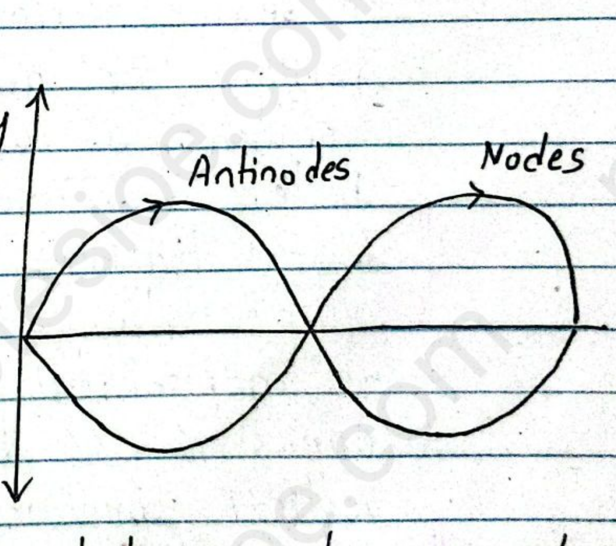
Now, $\text{Power}(P) = \frac{E}{t}$

$$= 2\pi^2 \rho F a^2 A v$$

$$\text{Intensity}(I) = \frac{P}{A} = 2\pi^2 \rho F a^2 v$$

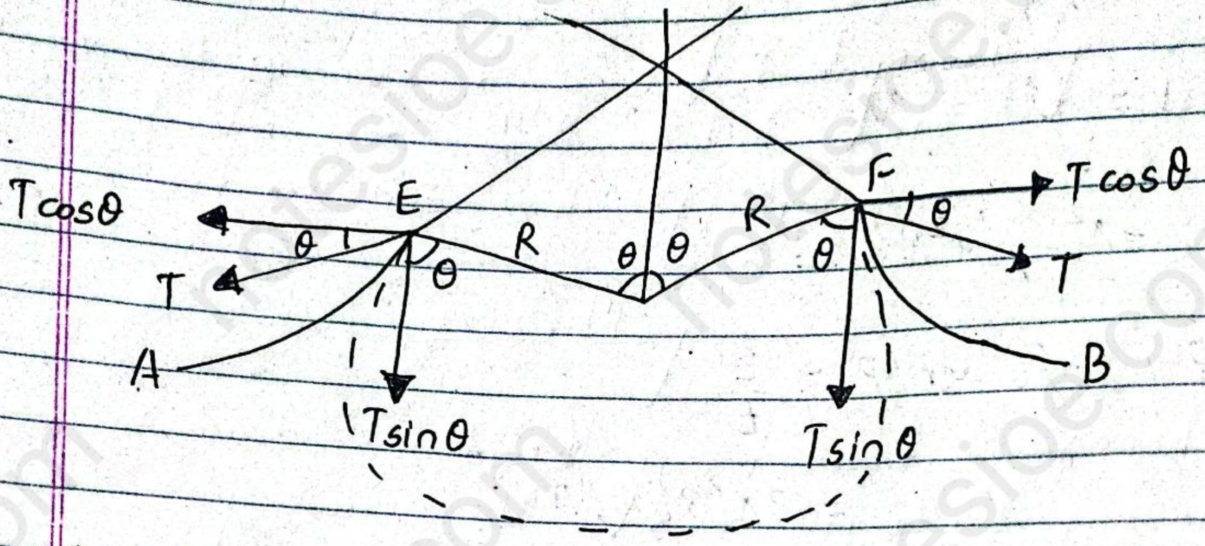
Standing or stationary wave

When two progressive waves of same altitude and frequency travelling through a medium with the same speed but in opposite direction superimposed to each other then stationary or standing wave is formed.



The wave is called stationary because it doesnot travel in both direction and all the particles of medium are permanently at rest. There is not flow of energy along the wave.

Velocity of transverse wave in a stretched string



consider a portion of AEFB of a stretched string with a transverse wave is travelling from left to right with velocity 'v'. consider a small string element EF of length (ΔL) forming an arc of a circle of radius R & subtending an angle 2θ at the centre of circle. A force with a magnitude is equal to the tension on the string 'T' pulls tangentially on this element at each end. The horizontal components are equal and opposite so they cancel each other and vertical compound gets added since they are along the same direction to give resultant tension.

ie. $F = 2(T\sin\theta)$

As θ is small, $\sin\theta \approx \theta$

$$F = T(2\theta)$$

$$F = T\frac{\Delta L}{R}$$

If μ is the linear mass density of the string and ΔM is the mass of the small element EF.

$$\Delta M = \mu \Delta L$$

The small amount has a centripetal acceleration.

$$a = \frac{v^2}{R}$$

Now, from Newton's 2nd law of motion,

$$F = ma$$

$$\frac{T\cancel{\Delta L}}{\cancel{R}} = (\mu \cancel{\Delta L}) \times \frac{v^2}{\cancel{R}}$$

or, $v^2 = \frac{T}{\mu}$

$$\boxed{\therefore v = \sqrt{\frac{T}{\mu}}}$$

which is required velocity of transverse wave in stretched string.